

Chemical Abundances in Symbiotic Stars

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ABSTRACT

We have carried out a study of the chemical abundances of ^1H , ^4He , ^{12}C , ^{13}C , ^{14}N , ^{15}N , ^{16}O , ^{17}O , ^{20}Ne and ^{22}Ne in symbiotic stars (SSs) by means of a population synthesis code. We find that the ratios of the number of O-rich SSs to that of C-rich SSs in our simulations are between 3.4 and 24.1, depending on the third dredge-up efficiency λ and the terminal velocity of the stellar wind $v(\infty)$. The fraction of SSs with *extrinsic* C-rich cool giants in C-rich cool giants ranges from 2.1% to 22.7%, depending on λ , the common envelope algorithm and the mass-loss rate. Compared with the observations, the distributions of the relative abundances of $^{12}\text{C}/^{13}\text{C}$ vs. $[\text{C}/\text{H}]$ of the cool giants in SSs suggest that the thermohaline mixing in low-mass stars may exist. The distributions of the relative abundances of C/N vs. O/N, Ne/O vs. N/O and He/H vs. N/O in the symbiotic nebulae indicate that it is quite common that the nebular chemical abundances in SSs are modified by the ejected materials from the hot components. Helium overabundance in some symbiotic nebulae may be relevant to a helium layer on the surfaces of white dwarf accretors.

Subject headings: binaries: symbiotic — accretion — stars: AGB — Galaxy: stellar content

1. Introduction

Symbiotic stars (SSs) are usually interacting binaries, composed of a cool star, a hot component and a nebula. The cool component is a red giant which is a first giant branch (FGB) or an asymptotic giant branch (AGB) star. The hot component is a white dwarf (WD), a subdwarf, an accreting low-mass main-sequence star, or a neutron star (Kenyon & Webbink 1984; Mürset et al. 1991; Yungelson et al. 1995; Iben & Tutukov 1996). In general, SSs are low-mass binaries with an evolved giant transferring materials to a hot WD companion which burns the accreted materials more or less steadily (Mikołajewska 2007). According to the observed characteristics of their specular spectra and photometries, SSs may stay in either the

quiescent phase or the outburst phase. During the quiescent phase, SSs are undergoing a stable hydrogen burning on the surfaces of the WD accretors. The outbursts may be due to either the thermonuclear outbursts on the surfaces of accreting WDs which are called as symbiotic novae (Tutukov & Yungelson 1976; Paczyński & Rudak 1980), or accretion-disk instabilities around a nearly steady burning WDs which are called as multiple outbursts (Mitsumoto et al. 2005; Mikołajewska 2007). Recent reviews of the properties of SSs can be found in Mürset & Schmid (1999) and Mikołajewska (2003, 2007).

The cool components usually have a high mass-loss rate in the SSs with WD accretors. Their stellar winds have the chemical abundances of the red giants. Parts of the stellar winds are ionized by the hot components. The abundance determinations with nebular diagnostic tools are possible (Schild, Boyle & Schmid 1992). SSs provide a good opportunity for measuring the abundances of the red giants. The hot components may support an additional high-velocity wind during the sym-

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biotic nova outbursts (Kenyon & Webbink 1984). High-velocity outflows have been observed in very broad emission lines in essentially all the symbiotic novae (Kenny & Taylor 2005). Livio & Truran (1994) analyzed classical nova abundances and concluded that enhanced concentrations of heavy elements are significant. Therefore, the abundances of the ejected materials from the hot components in SSs may be different from those of the cool giant stellar winds. However they are similar to those of the classical novae. In general, the hot components have high luminosity and effective temperature (They are similar to the central stars of planetary nebulae. See Figure 5 in Mürset et al. 1991) so that they can ionize the nebulae. In some eruptive SSs, the nebulae can also be ionized by the region where the winds from the hot and cold components collide (Willson et al. 1984). It is difficult to determine the origin of the nebulae in SSs. Based on emission line fluxes from C_{III}, C_{IV}, N_{III}, N_{IV} and O_{III}, Nussbaumer et al. (1988) found that the CNO abundance ratios of the nebulae in SSs are in the transition region from giants to supergiants and concluded that the nebula is mainly due to mass lost by the red giant. On the other hand, Nussbaumer & Vogel (1989) suggested that the flux ratios of the emission line depend on the relative abundances of the two winds in their study on Z And. Vogel & Nussbaumer (1992) found that CNO abundance ratios of PU Vul are of the characteristic of novae.

In short, SSs offer an exciting laboratory for studying novae, the red giants and the interaction of the two winds. The chemical abundances of the ejected materials from the hot components, the stellar wind of the cool components and the symbiotic nebulae are the key factor to understand them. Up to now, a series of observational data of the chemical abundances in SSs have been published. Nussbaumer et al. (1988), De Freitas Pacheco & Costa (1992), Costa & de Freitas Pacheco (1994) and Luna & Costa (2005) gave the chemical abundances of some symbiotic nebulae. Schild, Boyle & Schmid (1992) and Schmidt et al. (2006) showed the chemical abundances of the cool giants in several SSs. However, there are few theoretical studies about it. Recently, Kovetz & Prialnik (1997) and José & Hernanz (1998) carried out detailed numerical simulations for the chemical abundances of the novae with CO WD or

ONe WD accretors. This makes it possible to simulate the chemical abundances of the symbiotic novae. Groenewegen & de Jong (1993), Wagenhuber & Groenewegen (1998), Karakas, Lattanzio & Pols (2002), Izzard et al. (2004), Izzard (2004) and Marigo & Girardi (2007) developed thermally pulsing asymptotic giant branch (TP-AGB) synthesis by which the chemical abundances of the cool components can be computed. Lü, Yungelson & Han (2006) (hereafter Paper I) constructed the models for the SSs' populations. According to the above results or models, it is possible to set up a preliminary model for theoretical study on the chemical abundances of SSs.

In the present paper, the study of the chemical abundances of SSs are carried out by means of a population synthesis code. In § 2 we present our assumptions and describe some details of the modeling algorithm. In § 3 we discuss the main results and the effects of different parameters. § 4 contains the main conclusions.

2. Model

For the binary evolution, we use a rapid binary star evolution (BSE) code of Hurley et al. (2002). Below we describe our algorithm from several aspects.

2.1. Symbiotic Stars

Paper I carried out a detailed study of SSs. Comparing the observed distributions with the those predicted of the orbital periods and the hot component masses in SSs, it is worth to mention that there is remarkable disagreement between the observations and predictions.

Mikołajewska (2007) showed the distributions of the measured orbital period in about 70 SSs. The vast majority (~ 75%) of these systems have orbital periods shorter than 1000 days while Paper I predicted the distribution of longer orbital periods. The main reasons results from two aspects:

1. It is hard to measure long orbital periods. All the SSs with measured orbital periods in Belczyński et al. (2000) and Mikołajewska (2003) are S-type and about half of them are eclipsing binaries. Considering the amplitude of radial velocity changes or eclipses, it is easier to measure the short orbital

periods. About 15 percent of 27 SSs in Mikołajewska (2003) have longer orbital periods than 1000 days while it is about 25 percent in Mikołajewska (2007). There would be more SSs with long orbital periods with further detailed observations.

2. In Paper I, SSs are detached binary systems and the process of mass transfer results from the stellar wind of cool giants. To our knowledge, if the mass ratio of the components ($q = M_{\text{donor}}/M_{\text{accretor}}$) at onset of Roche lobe overflow (RLOF) is larger than a certain critical value q_c , the mass transfer is dynamically unstable and results in the formation of a common envelop in short time scale (about 100 years). The issue of the criterion for dynamically unstable RLOF q_c is still open. Han et al. (2001, 2002) showed that q_c depends heavily on the assumed mass-transfer efficiency. Paper I adopted q_c after Hurley et al. (2002) in which the mass-transfer efficiency is 1. In Paper I, q_c is usually smaller than 1.0. The observed $M_{\text{giant}}/M_{\text{WDs}}$ in SSs are between 2.0 and 4.0 and the predicted those in Paper I are from 1.0 to 4.0, which means that there should not be stable RLOF in SSs. However, Mikołajewska (2007) suggested that RLOF can be quite common in symbiotic binaries with orbital periods shorter than 1000 days. If this were true, the theoretical model of mass transfer should be advanced. But this is out of the scope of this paper.

The disagreement between the observed and the predicted distributions of orbital periods in Paper I resulted from observational biases and poor knowledge of the mass transfer mechanism.

Mikołajewska (2007) showed that most of SSs have WD masses less than $0.6 M_{\odot}$, while Paper I predicted the distribution of higher WD masses. The disagreement is due to the following:

1. For mass estimates, an orbital inclination of $i = 90^{\circ}$ or a limit to i (see table 2 in Mikołajewska (2003)) is typically assumed, hence the estimates are lower limits.
2. Paper I used the mass-loss law suggested by Vassiliadis & Wood (1993) for AGB stars. However, Mikołajewska (2007) suggested

that both the symbiotic giants and Miras have higher mass-loss rates than single giants or field Miras, respectively. Therefore, Paper I may have overestimated the hot component masses.

In short, due to poor knowledge of the mass loss from the giants and the mass transfer mechanism in SSs, Paper I can only crudely predict some observed characteristics. In this work, we accept still all the criteria and the concepts on SSs in it. Following Paper I, we assume that binary systems are considered as SSs if they satisfy the following conditions:

- The systems are detached.
- The luminosity of the hot component is higher than $10L_{\odot}$ which is the ‘threshold’ luminosity for the hot component of SSs as inferred by Mürset et al. (1991) and Mikołajewska & Kenyon (1992). This may be due to the thermonuclear burning (including novae outbursts, stationary burning and post-eruption burning).
- The hot component is a WD and the cool component is a FGB or an AGB star.

The liberation of gravitational energy by the accreted matter may make the luminosity of the component larger than $10L_{\odot}$. A detailed accretion model of SSs was discussed in Paper I. Here, we do not model it.

All symbiotic phenomena in our work are produced by the hydrogen burning on the WD surface. All SSs should stay in one of the following three phases:

- the stable hydrogen burning phase;
- the thermonuclear outburst phase (symbiotic nova);
- the declining phase after a thermonuclear outburst.

Paper I and the present paper do not simulate the accretion disk in SSs. As mentioned in the Introduction, the multiple outbursts due to the accretion-disk instabilities usually occur around nearly steady burning WDs (Mikołajewska 2007).

In this work, they can be included in the quiescent SSs which are undergoing the stable hydrogen burning.

SSs are complex binary systems. There are many uncertain physical parameters which can affect the population of SSs. Paper I showed that the numbers of SSs and the occurrences of symbiotic novae are greatly affected by the algorithm of common envelope evolution, the terminal velocity of stellar wind $v(\infty)$ and the critical ignition mass $\Delta M_{\text{crit}}^{\text{WD}}$ which is necessary mass accreted by WDs for a thermonuclear runaway. The structure factor of the stellar wind velocity α_{W} and an optically thick wind give a small uncertainty. In this work, we use the models of SSs in Paper I and discuss the effects of the algorithm of common envelope evolution, $v(\infty)$ and $\Delta M_{\text{crit}}^{\text{WD}}$ on the chemical abundances of SSs. Other subordinate parameters are the same as in the standard model of Paper I.

Common Envelope: For the common envelope evolution, it is generally assumed that the orbital energy of the binary is used to expel the envelope of the donor with an efficiency α_{ce} :

$$E_{\text{bind}} = \alpha_{\text{ce}} \Delta E_{\text{orb}}, \quad (1)$$

where E_{bind} is the total binding energy of the envelope and ΔE_{orb} is the orbital energy released in the spiral-in. Nelemans et al. (2000) suggested to describe the variation of the separation of components in the common envelopes by an algorithm. This algorithm is founded on the equation for the system orbital angular momentum balance which implicitly assumes the conservation of energy:

$$\frac{\Delta J}{J} = \gamma \frac{M_{\text{e}}}{M + m}, \quad (2)$$

where J is the total angular momentum and ΔJ is the change of the total angular momentum during common envelope phase. In the above formula, M and M_{e} are respectively the mass and the envelope mass of the donor, and m is the companion mass. Following Nelemans & Tout (2005) and Paper I, we call the formalism of Eq. (1) α -algorithm and that of Eq. (2) γ -algorithm, which are respectively simulated in different cases (see Table 1). We take the ‘combined’ parameter $\alpha_{\text{ce}} \lambda_{\text{ce}}$ as 0.5 for α -algorithm. λ_{ce} is a structure parameter that depends on the evolutionary stage of the donor.

For γ -algorithm, $\gamma = 1.75$.

$v(\infty)$: It is difficult to determine the terminal velocity of stellar wind $v(\infty)$. Winters et al. (2003) fitted the relation between the mass-loss rates and the terminal wind velocities derived from their CO(2-1) observation by

$$\log_{10}(\dot{M}/M_{\odot} \text{yr}^{-1}) = -7.40 + \frac{4}{3} \log_{10}(v(\infty)/\text{km s}^{-1}). \quad (3)$$

The mass-loss rate is given by the formulation of Vassiliadis & Wood (1993) or Blöcker (1995) and $v(\infty)$ can be obtained by Eq. (3). However, Eq. (3) is valid for \dot{M} close to $10^{-6} M_{\odot} \text{yr}^{-1}$. For a mass-loss rate higher than $10^{-6} M_{\odot} \text{yr}^{-1}$, Eq. (3) gives too high $v(\infty)$. Based on the models of Winters et al. (2000), we assume $v(\infty) = 30 \text{ km s}^{-1}$ if $v(\infty) \geq 30 \text{ km s}^{-1}$. In the standard model of Paper I, $v(\infty) = \frac{1}{2} v_{\text{esc}}$ where v_{esc} is the escape velocity. In the present paper, we also carry out various $v(\infty)$ s simulations.

$\Delta M_{\text{crit}}^{\text{WD}}$: The critical ignition mass of the novae depends mainly on the mass of accreting WD, its temperature and material accreting rate. Yungelson et al. (1995) gave the ‘constant pressure’ expression for $\Delta M_{\text{crit}}^{\text{WD}}$ as

$$\frac{\Delta M_{\text{crit}}^{\text{WD}}}{M_{\odot}} = 2 \times 10^{-6} \left(\frac{M_{\text{WD}}}{R_{\text{WD}}^4} \right)^{-0.8}, \quad (4)$$

where M_{WD} is the mass of WD accretor and R_{WD} is the radius of zero-temperature degenerate objects (Nauenberg 1972),

$$R_{\text{WD}} = 0.0112 R_{\odot} [(M_{\text{WD}}/M_{\text{ch}})^{-2/3} - (M_{\text{WD}}/M_{\text{ch}})^{2/3}]^{1/2}, \quad (5)$$

where $M_{\text{ch}} = 1.433 M_{\odot}$ and $R_{\odot} = 7 \times 10^{10} \text{ cm}$. Nelson et al. (2004) gave numerical fits to the critical ignition masses for novae models calculated by Prialnik & Kovetz (1995). In most simulations, we adopt Eq. (4) for $\Delta M_{\text{crit}}^{\text{WD}}$. However, in order to investigate the influences of the $\Delta M_{\text{crit}}^{\text{WD}}$ on our results, we also carry out a simulation for Eq. (A1) of Nelson et al. (2004) in which $\Delta M_{\text{crit}}^{\text{WD}}$ depends on the mass accretion rates and masses of WD accretors.

2.2. Chemical Abundances on the Surface of the Giant Stars

For a single star, three dredge-up processes and hot bottom burning in a star with initial mass higher than $4M_{\odot}$ may change the chemical abundances of the stellar surface. We accept the prescriptions of §3.1.2 and 3.1.3 in Izzard et al. (2004) for the first dredge-up during the first giant branch and the second dredge-up during early asymptotic giant branch (E-AGB). For the third dredge-up (TDU) and the hot bottom burning during the TP-AGB phase, we use the TP-AGB synthesis in Groenewegen & de Jong (1993), Karakas, Lattanzio & Pols (2002), Izzard et al. (2004), Izzard (2004) and Marigo & Girardi (2007). All details can be found in Appendix A. In the present paper, we give the chemical evolutions of ^1H , ^4He , ^{12}C , ^{13}C , ^{14}N , ^{15}N , ^{16}O , ^{17}O , ^{20}Ne and ^{22}Ne on the stellar surface.

SSs in our work are binary systems. The binary mass transfer can change the chemical abundances of the stellar surface. In binary systems, there are two ways to transfer mass: (i) accretion from the stellar wind material of a companion; (ii) Roche lobe overflow. BSE model contains a standard Bondi-Hoyle type wind accretion (Bondi & Hoyle 1944) and a conservative mass transfer during the stable Roche lobe overflow. After a star obtains ΔM from its companion, the chemical abundances of the stellar surface X_2 is given by

$$X_2^{\text{new}} = \frac{X_2^{\text{old}} \times M_2^{\text{env}} + X_1 \times \Delta M}{M_2^{\text{env}} + \Delta M}, \quad (6)$$

where M^{env} is the envelope mass and X_1 is the chemical abundances of its companion.

2.3. Chemical Abundances of the Ejected Materials from the Hot WD Accretors

The hot WD accretors eject materials from their surfaces. Kovetz & Prialnik (1997) and José & Hernanz (1998) showed detailed study on their chemical abundances during the thermonuclear outbursts. There is no direct observational evidence that SSs in the stable hydrogen burning phase and the declining phase lead to winds from the hot components. Theoretically, the hot companions have high luminosity and temperature during the above two phases. Some parts

of the materials on the surface of the accreting WDs may be blown off due to high luminosity (Iben & Tutukov 1996). To our knowledge, no literature refers to them in detail yet. However, their chemical abundances are possibly between those of the ejected materials during the thermonuclear outbursts and the stellar winds from the giant stars. In the present paper, we only give the chemical abundances of the ejected materials during the thermonuclear outbursts.

Based on the typical characteristics of the observed systems, nova outbursts are usually divided into symbiotic novae and classical novae. The classical novae can only last several days or weeks and the variation of their visual magnitudes are between -5 and -10. The symbiotic novae usually last several decades and the variation of their visual magnitudes are between -3 and -8. Thermonuclear runaways appear to be the most promising mechanism for classical novae (Kenyon 1986). Tutukov & Yungelson (1976) suggested that the above mechanism is applicable to symbiotic novae. The observational differences between the symbiotic novae and the classical novae may stem from the properties of the binary systems (Iben & Tutukov 1996). The physical nature of the classical novae and the symbiotic novae is identical. Therefore, we assume that the thermonuclear outbursts occurring in our binary systems selected for SSs are the symbiotic novae.

Numerical studies of the nova outbursts have been carried out by Shara, Prialnik & Kovetz (1993); Kovetz & Prialnik (1994); Prialnik & Kovetz (1995); Yaron et al. (2005). Kovetz & Prialnik (1997) published detailed multicycle calculations of the nova outbursts for CO WDs with mass ranging from 0.65 to 1.4 M_{\odot} . In their simulations, the element abundances of nova ejecta are affected by the four basic and independent parameters: C/O ratio in the accreting WD, its core temperature T_{WD} , the mass accreting rate \dot{M}_{WD} and the mass M_{WD} . In order to test the effect of the WD composition on the abundances of nova ejecta, Kovetz & Prialnik (1997) calculated the models with the accreting WDs composed of pure-C, pure-O and C/O=1, respectively. They found that the WD composition is not reflected in the abundances of ejecta. In an extensive study of close binary evolution, Iben & Tutukov (1985) showed that the AGB phase may be suppressed in

a close binary and a WD formed in such a system should have a ratio very close to unity. In our work, we assume that the CO WD composition is C/O=1. A WD temperature of 10^7 K corresponds to an age of 10^9 yr (Iben & Tutukov 1984). In fact, for most of SSs, the time interval between the formation of the WD and the beginning of the symbiotic stage may be longer than it, up to 10^{10} yr (Yungelson et al. 1995). Neglecting the effect of the nova outbursts on the temperature of WD accretor T_{WD} , we assume that it is 10^7 K. We select 10 models in Kovetz & Prialnik (1997) in which $T_{\text{WD}} = 10^7$ K and C/O ratio of the accreting WD is 1. Their element abundances are determined by the mass accreting rate \dot{M}_{WD} and M_{WD} . By a bilinear interpolation (Press et al. 1992) of 10 models in Kovetz & Prialnik (1997), the abundances of ^1H , ^4He , ^{12}C , ^{13}C , ^{14}N , ^{15}N , ^{16}O and ^{17}O in the nova ejecta are calculated. If \dot{M}_{WD} or M_{WD} in SSs are not in the range of the bilinear interpolation, they are taken as the most vicinal those in the 10 models.

Kovetz & Prialnik (1997) did not give the abundances of ^{20}Ne and ^{22}Ne . José & Hernanz (1998) gave the nucleosynthesis in the nova outbursts with CO and ONe WDs. In their calculations, the nova nucleosynthesis are affected by M_{WD} , \dot{M}_{WD} , the initial luminosity (or T_{WD}) and the degree of mixing between core and envelope. To test the effect of the WD mass, they carried out a number of simulations involving both CO WD ($M_{\text{WD}}=0.8, 1.0$ and $1.15 M_{\odot}$) and ONe ones ($M_{\text{WD}}=1.0, 1.15, 1.25$ and $1.35 M_{\odot}$). \dot{M}_{WD} is $2 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$ and their initial luminosity is $10^{-2} L_{\odot}$. The degree of mixing between the core and the envelope is a very uncertain parameter. José & Hernanz (1998) modeled three different mixing levels: 25%, 50% and 75%. Their results showed that higher mixing degree favors the synthesis of higher metal nuclei in ONe WDs. Following Starrfield et al. (1998), we adopt a 50% degree of the mixing. For the simulations of CO WDs in José & Hernanz (1998), we select the three nova models (The degree of the mixing is 50%) to calculate the abundances of ^{20}Ne and ^{22}Ne by the fitting formulae:

$$\begin{aligned} \log X(^{20}\text{Ne}) &= -3.206 + 0.14476 M_{\text{WD}}, \\ \log X(^{22}\text{Ne}) &= -2.57118 + 0.60784 M_{\text{WD}} - 0.33769 M_{\text{WD}}^2, \end{aligned} \quad (7)$$

where M_{WD} is in solar unit and the formulae agree

with the numerical results to within a factor of 1.1. In the above fits, the abundances of ^{20}Ne and ^{22}Ne depend weakly on M_{WD} . The range of CO WD mass M_{WD} is from ~ 0.5 to $1.4 M_{\odot}$. The abundances of ^{20}Ne calculated by Eq. (7) are between $\sim 10^{-3.13}$ and $10^{-3.00}$, and the abundances of ^{22}Ne are between $\sim 10^{-3.35}$ and $10^{-3.38}$. Therefore, we use Eq. (7) to calculate $X(^{20}\text{Ne})$ and $X(^{22}\text{Ne})$ of all symbiotic novae with the CO WDs. For the nova of ONe WD, we fit data of Table 3 in José & Hernanz (1998) for a 50% degree of mixing by:

$$\begin{aligned} \log X(^1\text{H}) &= -1.486 + 1.982 M_{\text{WD}} - 0.992 M_{\text{WD}}^2, \\ \log X(^4\text{He}) &= -0.839 - 0.103 M_{\text{WD}} + 0.197 M_{\text{WD}}^2, \\ \log X(^{12}\text{C}) &= -10.664 + 14.798 M_{\text{WD}} - 6.025 M_{\text{WD}}^2, \\ \log X(^{13}\text{C}) &= -14.691 + 22.517 M_{\text{WD}} - 9.607 M_{\text{WD}}^2, \\ \log X(^{14}\text{N}) &= 4.504 - 11.174 M_{\text{WD}} + 5.079 M_{\text{WD}}^2, \\ \log X(^{15}\text{N}) &= -2.196 - 2.313 M_{\text{WD}} + 2.404 M_{\text{WD}}^2, \\ \log X(^{16}\text{O}) &= -9.835 + 17.609 M_{\text{WD}} - 8.550 M_{\text{WD}}^2, \\ \log X(^{17}\text{O}) &= -14.492 + 22.095 M_{\text{WD}} - 9.366 M_{\text{WD}}^2, \\ \log X(^{20}\text{Ne}) &= -0.536 - 0.192 M_{\text{WD}}, \\ \log X(^{22}\text{Ne}) &= -21.20356 + 34.46454 M_{\text{WD}} - 15.9763 M_{\text{WD}}^2, \end{aligned} \quad (8)$$

which agree with the numerical results to within a factor of 1.3.

We neglect other elements because their abundances are much smaller than the above elements or their isotopes. All abundances in the nova ejecta are renormalized so that their sum is 1.0.

2.4. Symbiotic Nebulae

In SSs, there may be two winds. One comes from the cool giant and its chemical abundances are similar to those of the red giant envelope and its velocity is between ~ 5 and 30 km s^{-1} . The other may come from the hot components. The components during the symbiotic outbursts have the winds with a high velocity ($\sim 1000 \text{ km s}^{-1}$) although there is not a detailed description on the wind from the hot components during the quiescent phase and the declining phase (Kenyon 1986). Wind collision is inevitable in the symbiotic outbursts. Recently, Kenny & Taylor (2005, 2007) carried out a detailed study for the colliding winds in SSs. According to their simulations, the structure of the colliding winds (including temperature and density) is very complicated. Symbiotic nebulae should have a similar structure with the colliding winds at least during the symbiotic outbursts.

In order to avoid the difficulty of modeling the real symbiotic nebulae and study the nebular chemical abundances by population synthesis method, we assume roughly that the compositions of the symbiotic nebulae undergo two independent phases:

- The symbiotic nebula is mainly composed of the ejected materials from the hot WD when the thermonuclear runaway occurs and this phase last for t_{on} . The lasting time scale t_{on} on which the hot components are in a ‘plateau’ phase with high luminosity is given by Eq. (30) of Paper I. At this phase, the symbiotic nebula embodies the chemical characteristics of a nova.
- After t_{on} , the symbiotic nebula is mainly composed of the stellar wind materials from the cool giants until the next symbiotic nova occurs. During this phase, the symbiotic nebula embodies the chemical characteristics of the cool giants in our models. For stable hydrogen burning SSs, the symbiotic nebula is always composed of the stellar wind materials from the cool giants.

However, some symbiotic novae have helium WD accretors. To our knowledge, no literature refers to their chemical abundance. According to Paper I, the occurrence rate of the symbiotic novae with helium WDs is at most about $\frac{1}{75}$ of that with CO and ONe WDs. We do not consider SSs with helium WD accretors in this paper.

2.5. Basic Parameters of the Monte Carlo Simulation

We carry out binary population synthesis via Monte Carlo simulation technique in order to obtain the properties of SSs’ population. For the population synthesis of binary stars, the main input model parameters are: (i) the initial mass function (IMF) of the primaries; (ii) the mass-ratio distribution of the binaries; (iii) the distribution of orbital separations; (iv) the eccentricity distribution; (v) the metallicity Z of the binary systems.

We use a simple approximation to the IMF of Miller & Scalo (1979). The primary mass is generated using the formula suggested by

Eggleton et al. (1989)

$$M_1 = \frac{0.19X}{(1-X)^{0.75} + 0.032(1-X)^{0.25}}, \quad (9)$$

where X is a random variable uniformly distributed in the range $[0,1]$, and M_1 is the primary mass from $0.8M_{\odot}$ to $8M_{\odot}$.

For the mass-ratio distribution of binary systems, we consider only a constant distribution (Mazeh et al. 1992; Goldberg & Mazeh 1994),

$$n(q) = 1, \quad 0 < q \leq 1, \quad (10)$$

where $q = M_2/M_1$.

The distribution of separations is given by

$$\log a = 5X + 1, \quad (11)$$

where X is a random variable uniformly distributed in the range $[0,1]$ and a is in R_{\odot} .

In our work, the metallicity $Z=0.02$ is adopted. We assume that all binaries have initially circular orbits, and we follow the evolution of both components by BSE code, including the effect of tides on binary evolution (Hurley et al. 2002). We take 2×10^5 initial binary systems for each simulation. Since we present, for every simulation, the results of one run of the code, the numbers given are subject to Poisson noise. For simulations with 2×10^5 binaries, the relative errors of the numbers of the symbiotic systems in different simulations are lower than 5%. Thus, 2×10^5 initial binaries appear to be an acceptable sample for our study.

We assume that one binary with $M_1 \geq 0.8M_{\odot}$ is formed annually in the Galaxy to calculate the birthrate of SSs (Yungelson et al. 1994; Han et al. 1995a,b).

3. Results

We construct a set of models in which we vary different input parameters relevant to the chemical abundances of SSs. Table 1 lists all cases considered in this work. Many observational evidences showed that the terminal velocity of stellar wind $v(\infty)$ increases when a star ascends along the AGB (Olofsson et al. 2002; Winters et al. 2003; Bergeat & Chevallier 2005). In this work, we take $v(\infty)$ calculated by Eq. (3) as the standard terminal velocity of stellar wind. Case 1 is the standard model in this work. The results of SSs’ population are shown in Table 2.

Table 1: Parameters of the models of the population of SSs. The first column gives the model number. Columns 2, 3, 4 and 5 show the mass-loss rate \dot{M} , TDU efficiency λ , the minimum core mass for TDU and the inter-shell abundance, respectively. The detailed descriptions on the above parameters are given in Appendix A. The 6th 7th and 8th columns show the algorithm of the common envelope, the terminal velocity and the critical ignition mass, respectively. N04 in column 8 means Nelson et al. (2004).

Cases	\dot{M}	λ	M_c^{\min}	Inter-Shell Abundances	Common Envelope	$v(\infty)$	$\Delta M_{\text{crit}}^{\text{WD}}$
case 1	Eq.(A8)	Eq.(A4)	Eq.(A3)	Marigo & Girardi (2007)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	Eq.(3)	Eq.(4)
case 2	Eq.(A11)	Eq.(A4)	Eq.(A3)	Marigo & Girardi (2007)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	Eq.(3)	Eq.(4)
case 3	Eq.(A8)	0.5	Eq.(A3)	Marigo & Girardi (2007)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	Eq.(3)	Eq.(4)
case 4	Eq.(A8)	0.75	Eq.(A3)	Marigo & Girardi (2007)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	Eq.(3)	Eq.(4)
case 5	Eq.(A8)	Eq.(A4)	$0.58M_{\odot}$	Marigo & Girardi (2007)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	Eq.(3)	Eq.(4)
case 6	Eq.(A8)	Eq.(A4)	Eq.(A3)	Izzard et al. (2004)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	Eq.(3)	Eq.(4)
case 7	Eq.(A8)	Eq.(A4)	Eq.(A3)	Marigo & Girardi (2007)	$\gamma = 1.75$	Eq.(3)	Eq.(4)
case 8	Eq.(A8)	Eq.(A4)	Eq.(A3)	Marigo & Girardi (2007)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	$\frac{1}{2}v_{\text{esc}}$	Eq.(4)
case 9	Eq.(A8)	Eq.(A4)	Eq.(A3)	Marigo & Girardi (2007)	$\alpha_{\text{ce}}\lambda_{\text{ce}} = 0.5$	Eq.(3)	Eq.(A1) of N04

Table 2: Different models of the SSs' population. The first column gives the model number according to Table 1. Column 2 shows the birthrate of SSs in the Galaxy. From columns 3 to 5, the number of O-rich SSs, C-rich SSs and all SSs are given, respectively. In the 3rd and 4th columns, the numbers in parentheses mean the numbers of O-rich and C-rich SSs in which we neglect the effects of the mass transfer in binary systems on the chemical abundances on their surfaces, respectively. Columns 6 and 7 show the ratios of the number of SSs in the cooling phase and the stable hydrogen burning phase to their total number, respectively. Columns 8, 9 and 10 give the occurrence rates of SyNe (symbiotic novae) with the accreting CO WDs, ONe WDs and total rates. The number of SyNe with accreting CO and ONe WDs are shown in columns 11 and 12, respectively.

Cases	Birthrate of SSs (yr^{-1})	Number of SSs			$N_{\text{tcool}}/N_{\text{total}}$ $N_{\text{stable}}/N_{\text{total}}$		Occurrence rate of SyNe (yr^{-1})			Number of SyNe	
1	2	O-rich 3	C-rich 4	Total 5	$N_{\text{tcool}}/N_{\text{total}}$ 6	$N_{\text{stable}}/N_{\text{total}}$ 7	CO/ONe 8 9		Total 10	CO 11	ONe 12
case 1	0.119	4800 (4860)	870 (810)	5670	0.57	0.25	3.8	0.2	4.0	1000	4
case 2	0.118	5850 (6070)	1240 (1020)	7100	0.42	0.45	3.4	0.2	3.6	930	3
case 3	0.119	5340 (5390)	220 (170)	5560	0.59	0.24	3.7	0.2	3.9	940	3
case 4	0.119	4930 (4970)	840 (800)	5770	0.58	0.24	3.9	0.2	4.1	1000	4
case 5	0.111	4330 (4410)	1220 (1140)	5550	0.56	0.26	3.8	0.2	4.0	1000	4
case 6	0.119	4740 (4830)	930 (840)	5660	0.57	0.25	3.8	0.2	4.0	1000	4
case 7	0.160	7900 (7930)	1440 (1410)	9340	0.49	0.33	5.3	0.4	5.7	1700	12
case 8	0.072	2370 (2450)	700 (620)	3070	0.28	0.60	1.1	0.1	1.2	370	4
case 9	0.115	7830 (7900)	1790 (1720)	9620	0.75	0.15	9.8	0.4	10.2	1000	4

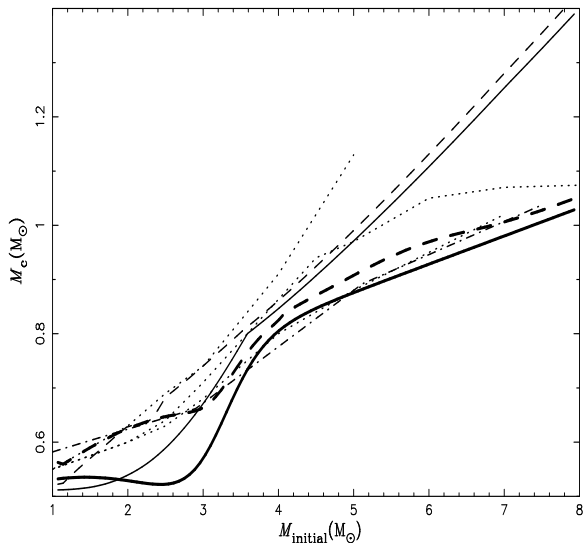


Fig. 1.— Initial-final mass relations. Thin and thick solid lines are the core masses at the first thermal pulse in Hurley et al. (2000) and in our work (See Eq. (A1)), respectively. Thin and thick dashed lines are the final masses in Hurley et al. (2000) and case 1 in this work, respectively. The dotted lines (from top to bottom) represent the relations from Girardi et al. (2000), Herwig (1995) and Weidemann (2000), respectively. The dot-dashed line represent the relation from Han et al. (1994).

3.1. Galactic Birthrate and the Number of SSs

First, we discuss the gross properties of the modeled population of SSs as those in Paper I. As Table 2 shows, the Galactic birthrate of SSs may range from 0.072 (case 8) to 0.160 (case 7) yr^{-1} and their number is from about 3070 (case 8) to 9620 (case 9). The occurrence rate of the symbiotic novae is between 1.2 (case 8) and 10.2 (case 9) yr^{-1} and the number of the symbiotic novae is from about 370 (case 8) to 1700 (case 7). The contribution of the symbiotic novae with ONe WD accretors to the occurrence rate is negligible in all cases, which is greatly different from that in Paper I. The main reason is the different stellar evolutions adopted during the TP-AGB phases.

All assumptions in cases 1 and 8 are respectively the same as those in case 7 and the standard model of Paper I except the stellar evolutions during the TP-AGB phase. The greatest difference of the TP-AGB phase between this work and Paper I is the initial-final mass relations. It is well known that the relation is still very uncertain. Fig. 1 shows the initial-final mass relations in different literatures. Using the mass-loss rate of Vassiliadis & Wood (1993) and assuming the classical core mass luminosity relation, Weidemann (2000) gave an initial-final mass relation which is plotted in Fig. 1. The relation for the more massive stars in Girardi et al. (2000) is steep because TDU and hot bottom burning were not included. By applying new observational data (including NGC2168, NGC2516, NGC2287 and NGC3532) and improved theory (stellar evolution calculations including new opacities and main sequence overshoot; the extensive mass grid of WD cooling sequences including models with thick and thin He- and H-layer), Herwig (1995) gave an initial-final mass relation which has a less slope than that in Girardi et al. (2000) for the more massive stars. Based on new stellar evolution calculations (including TDU and hot bottom burning) and new observational data, Weidemann (2000) showed an initial-final mass relation which has lower final mass than that of Herwig (1995) in the more massive stars. Using a criterion for envelope ejection in AGB or FGB stars, Han et al. (1994) obtained an initial-final mass relation which is very similar to that in Weidemann (2000). Paper I accepted the description of Hurley et al. (2000) on

TP-AGB evolution. The thin solid line and the dashed line in Fig. 1 represent the core mass at the first thermal pulse $M_{c,1tp}$ and the initial-final mass relations in Hurley et al. (2000), respectively. In this paper, we obtain the final mass by the synthesis TP-AGB evolutions (Detailed descriptions can be seen in Appendix A). The $M_{c,1tp}$ (See Eq. (8) in Karakas, Lattanzio & Pols 2002) and the final mass are shown by the thick solid and dashed lines in Fig. 1, respectively. The relation of Hurley et al. (2000) is steeper than ours for $M_{\text{initial}} > 4M_{\odot}$ (especially $M_{\text{initial}} > 6M_{\odot}$) and gives a higher WD's mass than ours. In Paper I, we assumed that the ONe WD originates from a star with initial mass being between $6.1 M_{\odot}$ and about $8 M_{\odot}$ (Pols et al. 1998). In this work, we still keep the above assumption. The masses of ONe WDs in this work are much lower than those in Paper I. A small mass of the accreting WD is unfavorable for producing the symbiotic phenomenon. Compared with case 7 and the standard model in Paper I, the birthrate, the number of SSs and the symbiotic novae decrease in cases 1 and 8. Especially, the occurrence rate of the symbiotic novae with accreting ONe WDs in case 1 is $\sim 3\%$ of that in case 7 of Paper I, and in case 8 it is $\sim 5\%$ of that in the standard model of Paper I. Therefore, the initial-final mass relation has great effects on the SSs' population and especially on the symbiotic novae.

Using the observational data in the last 25 years, Iben & Tutukov (1996) estimated that the occurrence rate of symbiotic novae is $\sim 1 \text{ yr}^{-1}$ within a factor of 3. As Table. 2 shows, the occurrence rate in case 1 is ~ 4 and close to the occurrence rate estimated by Iben & Tutukov (1996). Under the same conditions, case 7 in Paper I gave a higher occurrence rate of $\sim 11.9 \text{ yr}^{-1}$. Obviously, an steep initial-final mass relation like that in Hurley et al. (2000) overestimates the occurrence rate of symbiotic novae. This indicates that the initial-final mass relations from Han et al. (1994), Weidemann (2000) and the present paper are more reliable than steep ones. However, due to the complicity of SSs, the above point of view needs supports from further theoretical study and more observational evidence.

3.2. The Effects of Input Parameters

In this section, we discuss the effects of the input physical parameters on the SSs' population and the ratio of the number of C-rich SSs in which the cool companions are C-rich giants to that of O-rich SSs in which the cool companions are O-rich giants. In Paper I, we analyzed the effects of the physical parameters (the common envelope algorithm, the terminal velocity of the stellar wind $v(\infty)$ and the critical ignition mass of the novae $\Delta M_{\text{crit}}^{\text{WD}}$) on the SSs' population. They have a great effect on the SSs' population in our simulations, which is similar to those in Paper I. Table 2 shows, the *common envelope algorithm* and $\Delta M_{\text{crit}}^{\text{WD}}$ have weak effects on the ratio of the number of O-rich to that of C-rich SSs while the $v(\infty)$ in case 8 takes an uncertainty within a factor of about 1.6. With stars ascending along the AGB, the $v(\infty)$ in case 8 decreases while more C-rich giants are formed. A small $v(\infty)$ is favor for the formation of SSs. Therefore, the ratio of O-rich to C-rich SSs in case 8 decreases. The effects of other physical parameters are following.

Mass-loss rate: Compared with Vassiliadis & Wood (1993)(see Eq. (A8)), the mass loss rate of Blöcker (1995)(see Eq. (A11)) greatly depends on stellar luminosity. When the cool giants ascend along the AGB, the luminosities gradually increase. Therefore, case 2 gives a higher mass-loss rate in the later phase during the AGB, which can increase the SSs' number of the stable hydrogen burning while decrease the SSs' number of the thermonuclear runaways. In general, C-rich giants are formed at a later phase of the AGB by the third dredge-up. The mass-loss rate in Blöcker (1995)(See Eq. (A11)) enhances the number of SSs and the number ratio of C-rich to O-rich SSs but decreases the occurrence rate of the symbiotic novae and their number. However, the effect is weak.

TDU efficiency λ : Comparing cases 1, 3 and 4, we find that λ has a great effect on the ratio of O-rich to C-rich SSs' population. The effect of λ on stellar evolution has mainly two aspects, *i.e.* the core mass evolution and the changes of the chemical abundances in stellar envelope. The average dredge-up efficiency in case 1 is higher than that in case 3 ($\lambda = 0.5$) and approximated to that in case 4 ($\lambda = 0.75$). In cases 1, 3 and 4, the SSs' populations are almost the same. However, the

number ratio of O-rich to C-rich SSs in case 3 is 4 times more than the ratio in cases 1 or 4.

The minimum core mass for TDU M_c^{\min} : For the same initial masses, the M_c^{\min} of Karakas, Lattanzio & Pols (2002) (See Eq. (A3)) is higher than $0.58M_\odot$. In case 1, a star with an initial mass higher than $\sim 2.0M_\odot$ can undergo the third dredge-up, but only $1.5M_\odot$ in case 5. Therefore, C-rich SSs are more easily produced in case 5. Its effect is within a factor about 1.5.

The inter-shell abundances: As cases 1 and 6 in Table 2 show, the two different inter-shell abundances have no effect on the SSs' populations. In Izzard et al. (2004), ^{16}O abundance of the inter-shell region is lower than that in Marigo & Girardi (2007). The number of C-rich SSs in case 6 increases. However, the effect of the inter-shell abundances on ratio of O-rich to C-rich SSs is very weak.

In short, the mass-loss rate and the inter-shell abundances have very weak effect on the SSs' population and the ratio of O-rich to C-rich SSs. The TDU efficiency λ has a very weak effect on the SSs' population, while the number ratio of O-rich to C-rich SSs have a great dependence on λ .

3.3. C-rich SSs

Belczyński et al. (2000) presented a catalogue of SSs which includes 188 SSs as well as 28 objects suspected of being SSs. According to the spectral types of the cool components listed by the catalogue, one can find out 5 C-rich SSs in 176 Galactic SSs and the number ratio of O-rich to C-rich SSs is about 35. In our simulations, from cases 1 to 9, the number ratios of O-rich to C-rich SSs are about 5.5, 4.9, 24.1, 5.8, 3.6, 5.2, 5.6, 3.4 and 4.3, respectively. Our results are lower than the above observational ratio. In all cases, the ratio of case 3 with a low $\lambda=0.5$ is the closest to the ratio observed in Belczyński et al. (2000). However, according to the typical TP-AGB synthesis (Groenewegen & de Jong 1993; Karakas, Lattanzio & Pols 2002; Izzard et al. 2004; Marigo & Girardi 2007), $\lambda=0.5$ may be too low for the solar-like stars. In the solar neighborhood, the number ratio of M-type giants to C-type giants is ~ 5 in Groenewegen (2002), which is close to our results except for case 3. We consider that the small fraction of the observed C-rich giants in SSs may result from two respects: (i) The C-

rich giants may have high terminal velocity of the stellar wind $v(\infty)$. When we use the fit of $v(\infty)$ in Winters et al. (2003), we should note that the sample of Winters et al. (2003) has 65 sources but only two of them are C-rich giants. Eq. (3) may be unsuitable for the C-rich giants. Comparing cases 1 and 8, one can find that $v(\infty)$ has a great effect on the number ratio of O-rich to C-rich SSs. We may have underestimated $v(\infty)$ of C-rich giants. (ii) The cool giants in SSs have high mass-loss rates. In cases 1 and 2, we use the mass-loss rates of Vassiliadis & Wood (1993) and Blöcker (1995), respectively. The average mass-loss rate of the former is higher than that of the later during TP-AGB when the TDU occurs. The higher the mass-loss rate is, the more quickly the envelope mass decreases. According to Eq. (A2) in the present paper, the interpulse period τ_{ip} increases with the envelope mass decreasing. A long τ_{ip} reduces the TDU progressive number and the TDU efficiency λ . The large mass-loss rate is unfavorable for the formation of the carbon stars, which can be seen by comparing the number ratio of O-rich to C-rich SSs in case 1 with that in case 2. Mikołajewska (2007) suggested that both the symbiotic giants and Miras have higher mass-loss rates than single giants or field Miras, respectively. Therefore, the predicted number ratio of O-rich to C-rich SSs lower than the observational value may result from our underestimating the $v(\infty)$ of C-rich giants and the mass-loss rates of the cool giants in SSs.

Based on the formation channels, C-rich stars are classed into two types. The *intrinsic* C-rich stars mean that the carbon observed in the atmosphere of the TP-AGB stars results from the third dredge-up. The *extrinsic* C-rich stars are the giants polluted by carbon-rich matter from the former TP-AGB companion. In this work, we consider the effects of the mass transfer in binary systems on the chemical abundances on their stellar surfaces (See §2.2). In order to check the above effects, we also carry out the simulations in which the mass transfer between the two components can not vary the abundances on their surfaces. The results are shown in the parentheses of columns 3 and 4 of Table. 2. The ratios of SSs with the *extrinsic* C-rich giants to the total C-rich SSs from case 1 to case 9 are 6.8%, 17.7%, 22.7%, 4.7%, 6.6%, 9.7%, 2.08%, 11.4% and 3.9%,

respectively. The ratio is sensitive to the efficiency of TDU λ , the common envelope algorithm and the mass-loss rate. According to the discussions in §3.2, we know that a high mass-loss rate occurs in case 2 when carbon abundance is increased by the TDU. The *extrinsic* C-rich giants in case 2 are formed more easy than those in case 1. A low λ in case 3 can prevent the formation of C-rich stars including *intrinsic* and *extrinsic* C-rich stars. The high ratio in case 3 results from a small number of C-rich stars. Having undergone the α -algorithm common envelope evolution, the orbital period of binary system becomes to be several percent of that before common envelope evolution. With γ -algorithm in case 7, post-common-envelope systems are wider than with the α -algorithm in case 1 and this facilitates a symbiotic phenomenon, allowing more stars to evolve further along the FGB or AGB before the second RLOF (Paper I). In general, post-common-envelope systems hardly form the *extrinsic* C-rich stars but can form *intrinsic* C-rich stars. Therefore, the ratio of SSs with the *extrinsic* C-rich giants to the total C-rich SSs in case 7 is very low. In most cases, the fraction of *extrinsic* C-rich SSs in total C-rich SSs is lower than 10%. This indicates that the pollution of the carbon rich materials from the former TP-AGB companions on the cool components is weak so that most of C-rich giants in SSs are *intrinsic* C-rich stars. Mikołajewska (2007) showed that all C-rich giants observed in SSs are *intrinsic* C-rich stars, which is in agreement with our results.

3.4. $^{12}\text{C}/^{13}\text{C}$ vs. $[\text{C}/\text{H}]$ of the Cool Giants in SSs

Using the infrared spectra of SSs, Schild, Boyle & Schmid (1992) and Schmidt & Mikołajewska (2003) gave the C abundance and $^{12}\text{C}/^{13}\text{C}$ isotopic ratio of the cool giants in 13 SSs with ± 0.3 errors. Using high-resolution near-infrared spectra and the method of the standard local thermal equilibrium analysis and atmospheric models, Schmidt et al. (2006) calculated the abundance of the symbiotic star CH Cyg with the errors less than 0.3 dex for all the elements. Their observational data are shown in panels (10) of Fig. 2, where $[\text{C}/\text{H}]$ means the relative abundances to the solar, that is, $[\text{C}/\text{H}] = \log \text{C}/\text{H} - \log \text{C}_{\odot}/\text{H}_{\odot}$. The left panel of Fig. 2 shows that $^{12}\text{C}/^{13}\text{C}$ and $[\text{C}/\text{H}]$ of the cool giants in SSs are much higher than those

from the observations. Schild, Boyle & Schmid (1992) used the synthetic spectra calculated for M giants by Lazaro et al. (1991) in which they analyzed M giants and obtained a low mean $[\text{C}/\text{H}] = -0.64 \pm 0.29$. Lazaro et al. (1991) suggested that the standard mixing named as the first dredge-up (See Iben & Renzini 1983) is insufficient to explain atmospheric abundances in M giants. In our work, we use the standard mixing which only includes the first dredge-up. ^{12}C abundance on stellar surface is reduced by approximately 30% after the first dredge-up during the FGB (Iben & Renzini 1983).

Gratton et al. (2000) determined Li, C, N, O, Na and Fe abundances and $^{12}\text{C}/^{13}\text{C}$ isotopic ratios of 62 metal-poor field giants. They suggested that there are two distinct mixing stages along the red giant branch evolution of small mass field stars: (i) The first dredge-up, that is, the standard mixing; (ii) The second mixing episode which is also called the thermohaline mixing in Charbonnel & Zahn (2007) when the giants have a brighter luminosity. The thermohaline mixing can decrease ^{12}C abundances. Charbonnel & Zahn (2007) calculated several evolution models of a $0.9M_{\odot}$ star and obtained the abundances which are consistent with the observational data in Gratton et al. (2000). Recently, Eggleton et al. (2006, 2007) confirmed a possible mechanism for such non-canonical mixing. Using 3D-modeling of a low-mass star at the tip of red giant branch, they found that the molecular inversion can lead to an efficient mixing and Pop I stars between 0.8 and $2.0 M_{\odot}$ develop $^{12}\text{C}/^{13}\text{C}$ ratios of 14.5 ± 1.5 . The average of $[\text{C}/\text{H}]$ in Schild, Boyle & Schmid (1992) is about -0.54 and the average $^{12}\text{C}/^{13}\text{C}$ is about 16. It range in our simulations is between about -0.05 (case 3) and 0.15 (case 4) for $[\text{C}/\text{H}]$ and between about 23.4 (case 3) and 37.2 (case 5) for $^{12}\text{C}/^{13}\text{C}$, respectively. The main reason of the above disagreement is that the thermohaline mixing is not included in our simulation. In order to show the effects of the thermohaline mixing, we artificially decrease ^{12}C abundance of all giants with initial masses $\leq 2M_{\odot}$ by a factor of 2.5 after the first dredge-up. The results are given in the right panel of Fig. 2. The averages of $[\text{C}/\text{H}]$ are between -0.31 (case 3) and -0.102 (case 5) and the averages of $^{12}\text{C}/^{13}\text{C}$ are between 12.3 (case 3) and 19.5 (case 5). Our results are then close to the observations

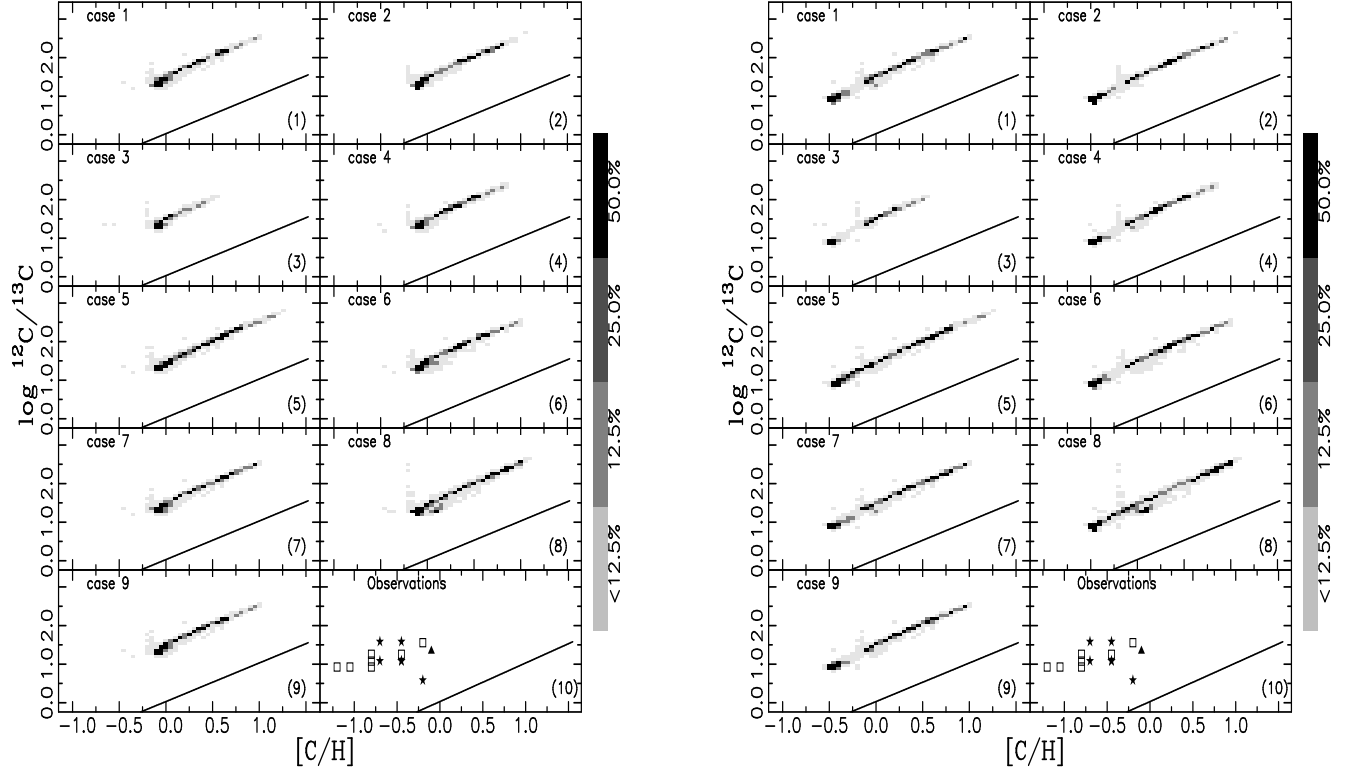


Fig. 2.— Gray-scale maps of the chemical abundance ratios of $\log^{12}\text{C}/^{13}\text{C}$ versus $[\text{C}/\text{H}]$. The gradations of gray-scale correspond to the regions where the number density of systems is, respectively, within $1 - 1/2$, $1/2 - 1/4$, $1/4 - 1/8$, $1/8 - 0$ of the maximum of $\frac{\partial^2 N}{\partial \log^{12}\text{C}/^{13}\text{C} \partial \log [\text{C}/\text{H}]}$, and blank regions do not contain any stars. Solid lines mean $\log^{12}\text{C}/^{13}\text{C} = [\text{C}/\text{H}]$. The left panel of Fig. 2 shows the chemical abundance ratios of the cool giants in SSs. The right panel of Fig. 2 gives those of the cool giants in SSs, where ^{12}C abundance of all giants with initial masses $\leq 2M_{\odot}$ is artificially decreased by a factor of 2.5 after the first dredge-up. In the panel of observations, the filled big-stars, the empty squares and triangle represent the observational values in Schild, Boyle & Schmid (1992), those in Schmidt & Mikołajewska (2003) and those in Schmidt et al. (2006), respectively.

of Schild, Boyle & Schmid (1992).

We believe that the thermohaline mixing can occur and give significantly different results although the observational samples of $^{12}\text{C}/^{13}\text{C}$ vs. $[\text{C}/\text{H}]$ of the cool giants in SSs is small. However, due to poor knowledge on the thermohaline mixing, we can not carry out a detailed simulation. In the following sections, we do not discuss its effects any more.

3.5. Chemical Abundance of Symbiotic Nebulae

In this subsection, we discuss the chemical abundances of the symbiotic novae. In order to clarify the nature of the symbiotic nebulae, we need the observational abundances of novae and planetary nebulae. In this paper, the nova abundances come from Livio & Truran (1994) in which they analyzed the abundances of 18 classical novae with a factor of ~ 2 -4 uncertainty. The planetary nebulae abundances are obtained from Pottasch & Bernard-Salas (2006) in which they gave the chemical abundance of 26 planetary nebulae within a 30% uncertainty in the abundance determination.

3.5.1. C/N vs. O/N

Based on UV data, Nussbaumer et al. (1988) presented C/N and O/N abundance ratios of 24 symbiotic nebulae. The errors in the logarithmic (basis of 10) abundance ratios remain within 0.18. In their sample, there are at least three symbiotic novae V1016 Cyg, HM Sge and HBV 475. V1016 Cyg and HBV 475 had their outbursts around 1965 while HM Sge brightened by 5 mag in 1975. Schmid & Schild (1990) calculated their abundance ratios within an error of 30%. Their C/N and O/N abundance ratios are different from those in Nussbaumer et al. (1988) within a factor of 30%. According to *IUE* observations, HM Sge is still evolving towards higher excitation whereas V1016 Cyg and HBV 475 seem to have reached their maximum excitation (Nussbaumer & Vogel 1990). The outburst of PU Vul began late in 1977. Vogel & Nussbaumer (1992) gave the C/N and O/N abundances ratios in the early nebular phase. The observational results and our simulated results are shown in Fig. 3.

The left and right panels (1)—(9) of Fig. 3 give

the distribution of the relative CNO abundances of the cool companions and the ejected materials from the hot components during symbiotic novae, respectively. Panels (10) in Fig. 3 show the observational ratios of C/N vs. O/N of symbiotic nebulae, novae and planetary nebulae.

The left panel of Fig. 3 shows that we can cut the abundance ratios of C/N vs. O/N of the cool companions in SSs into three regions. i) The region of $\text{C/N} > \text{O/N}$ represents that the cool companions have undergone the deep TDU and have not undergone the hot bottom burning. The initial masses of the cool companions should be between about 2.0 or 1.5(case 5) M_{\odot} and 4.0 M_{\odot} . Their abundances are similar to those of C-stars and planetary nebulae; ii) The top region of $\text{C/N} < \text{O/N}$ represents that the cool companions have undergone inefficient TDU or have not undergone TDU. These stars have initial masses lower than 2.0 or 1.5 (case 5) M_{\odot} . Their abundances are similar to those of M-stars; iii) The bottom region of $\text{C/N} < \text{O/N}$ represents that the cool companions undergo TDU and the hot bottom burning which turns ^{12}C to ^{14}N . Their initial masses are higher than 4.0 M_{\odot} .

In the right panel of Fig. 3, the distribution of C/N vs. O/N of the ejected materials from the hot companions during symbiotic novae is composed of two regions. The top region represents symbiotic novae with accreting ONe WDs. The bottom region represents those with accreting CO WDs. Compared with the observations, our results agree with those in Livio & Truran (1994).

From the left panel of Fig. 3, we find that the relative CNO abundances of the cool companions in our simulations are close to those of the observational symbiotic nebulae. However, comparing more carefully, one can find that many observed SSs lie in the transition from normal M giants with initial mass lower than 4 M_{\odot} (no hot bottom burning) to the super giants with higher initial mass. This agrees with Nussbaumer et al. (1988). If we compare Fig. 3 with the observations, we find that the abundance ratios of most of observational symbiotic nebulae are between those of the cool giants and the ejected materials from the hot companions during symbiotic novae. This suggests that the compositions of the symbiotic nebulae may be modified by the ejected materials from the hot companions.

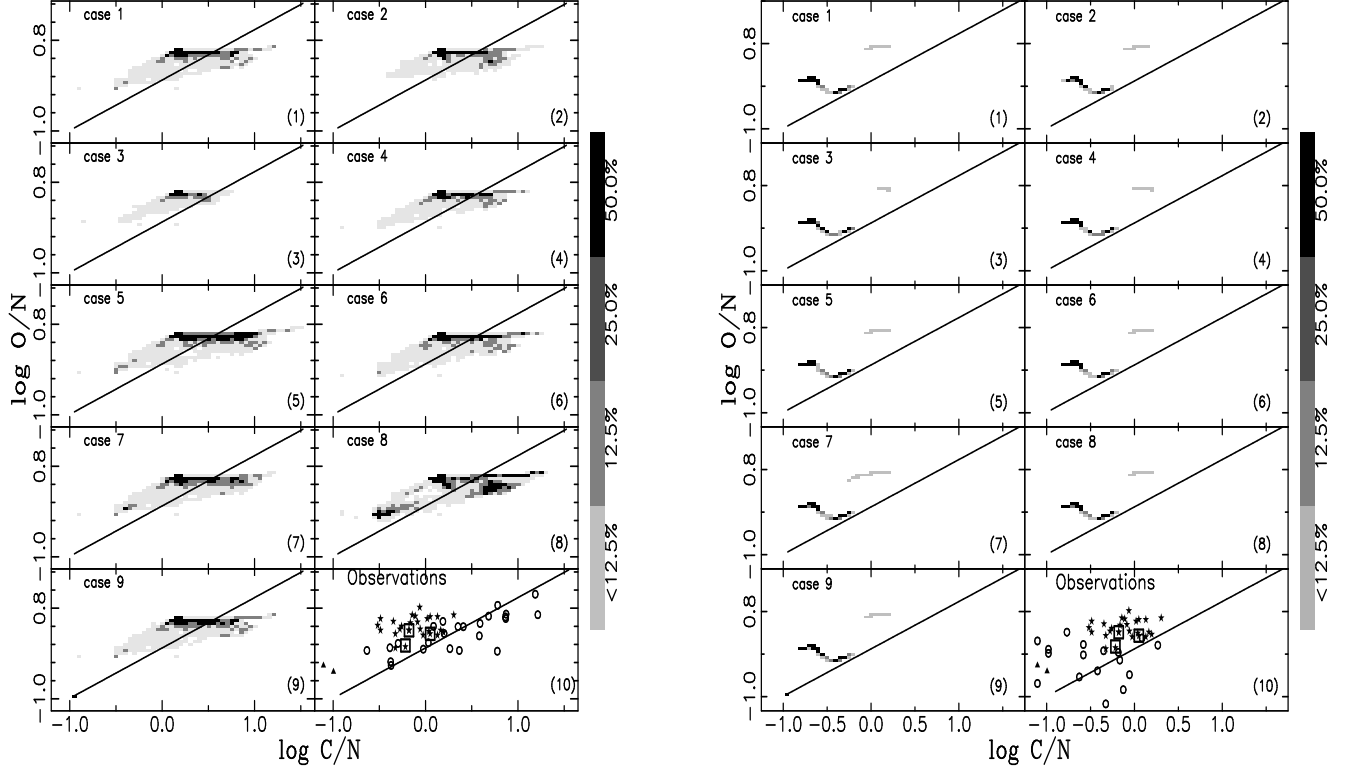


Fig. 3.— Gray-scale maps of the chemical abundance ratios of $\log C/N$ versus $\log O/N$. The gradations of gray-scale correspond to the regions where the number density of systems is, respectively, within $1 - 1/2$, $1/2 - 1/4$, $1/4 - 1/8$, $1/8 - 0$ of the maximum of $\frac{\partial^2 N}{\partial \log C/N \partial \log O/N}$, and blank regions do not contain any stars. Solid lines mean $\log C/N = \log O/N$. The left and right panels of Fig. 3 show the chemical abundances of the giants during the symbiotic stage and the ejected materials from the hot companions during the symbiotic novae, respectively. In the panels of observations, the filled big-stars, the filled triangles and the squares represent the observational values of the symbiotic nebulae in Nussbaumer et al. (1988), those of PU Vul in Vogel & Nussbaumer (1992) (including C_{IV} and N_{IV} or excluding them) and those of Cyg, HM Sge and HBV 475 in Schmid & Schild (1990), respectively. The circles in the left and right panels (10) of Fig. 3 represent the observational values of the planetary nebulae in Pottasch & Bernard-Salas (2006) and the novae in Livio & Truran (1994), respectively.

As the panels (10) of Fig. 3 show, the abundance ratios of the symbiotic novae Pul Vul and HM Sge are like those of the novae while the ratios of V1016 Cyg and HBV 475 are similar to those of the giants. Pul Vul and HM Sge are still staying on the early phase of the nova outbursts whereas V1016 Cyg and HBV 475 may be staying on the decline phase. Vogel & Nussbaumer (1992) predicted that the chemical abundances of Pul Vul would become closer to those of red giants in the future. Therefore, we believe that the compositions of the symbiotic nebulae have been modified by the ejected materials from the hot companions during symbiotic novae. Can the modification only last t_{on} like our assumption in §2.4? According to Table 2, most of SSs are in the cooling phase of the symbiotic novae and the stable hydrogen burning phase. The symbiotic novae are only 18.3% (case 7) and 10% (case 9) of total SSs. However, most of the observed symbiotic nebulae have mixing CNO abundances, which means that the above modification should be common in all SSs. We showed the C/N and O/N abundance ratios of several typical SSs which stay in different phases in order to support the above the view of point.

Declining phase: AG Pegasi is currently in decline from a symbiotic nova outburst which began in about 1850. Based on multi-shell radio emission from AG Pegasi in Kenny, Taylor & Seaquist (1991), the nebula can be divided into three parts. The outer nebula is thought to be the remnant of the 1850 outburst. The intermediate nebula may represent the pre-eruption mass loss from the cool component. Using a new colliding winds models including the effects of orbital motion, Kenny & Taylor (2007) suggested that the inner nebula originates from the colliding winds and has an expansion velocity of about 50 km s^{-1} . It takes about 1000 yr for the inner nebula to reach the region of the outer nebula. In general, the timescale of the cooling phases t_{cool} is several hundred years. Therefore, during the cooling phase after the thermonuclear runaways, the ejected materials from the hot components have an effect on the chemical compositions of the symbiotic nebulae.

Multiple outbursts: Sokoloski et al. (2006) discussed the outburst of symbiotic system Z And occurring in 2000-2002. They believed that the outburst in 2000-2002 results from the enhanced

shell burning triggered by a disk instability. During the burst occurring in 2000-2002, a shell of material blown from the surface of the WD is found. The blown material may result from an optically thick wind. Both Z And and CH Cyg are SSs which have undergone multiple outbursts. Since 1964, CH Cyg has displayed a number of outbursts. Schmidt et al. (2006) showed both the photospheric features of the cool giant and the nebular emission lines in CH Cyg. The average C/N and O/N ratios of the cool giants in CH Cyg are respectively 1.6 and 4.0, which are typical abundance ratios of M giants. The average ratios of the symbiotic nebulae in CH Cyg are respectively 0.57 and 2.0, which are similar to those of super giant stars and are between those of giants and novae. The significant discrepancy of C/N and O/N ratios derived from these two components suggest that the chemical abundances of the nebulae in CH Cyg are modified by the ejected materials from the hot components.

Quiescent phase: Sy Mus lacks any outburst activity and it is in the stable hydrogen burning phase (Mikołajewska 2007). Nussbaumer et al. (1988) showed that the C/N and O/N ratios in its nebulae are 0.97 and 1.9, respectively. These ratios are similar to those of super giant stars and are between those of giants and novae. This means that the chemical abundances of the nebulae of Sy Mus in the quiescent phase are modified by the ejected materials from the hot components.

Therefore, we suggest that it is quite common in SSs that the chemical abundances of the symbiotic nebulae are modified by the ejected materials from the hot WD surfaces.

3.5.2. Ne/O vs. N/O

According to optical data, Luna & Costa (2005) calculated He abundance and the relative abundances N/O, Ne/O and Ar/O of 43 symbiotic nebulae. We first discuss Ne/O vs. N/O. The mean errors in the logarithmic (basis of 10) abundance ratios are $\sim \pm 0.1$. De Freitas Pacheco & Costa (1992) showed the ratios of N/O and Ne/O in 5 symbiotic nebulae within an error of 0.15. Using the observational data, we note that the observational results for same SSs in different literature are quite different. The ratios of N/O and Ne/O of V1016 Cyg in De Freitas Pacheco & Costa (1992) differ by a factor of 4 from those in

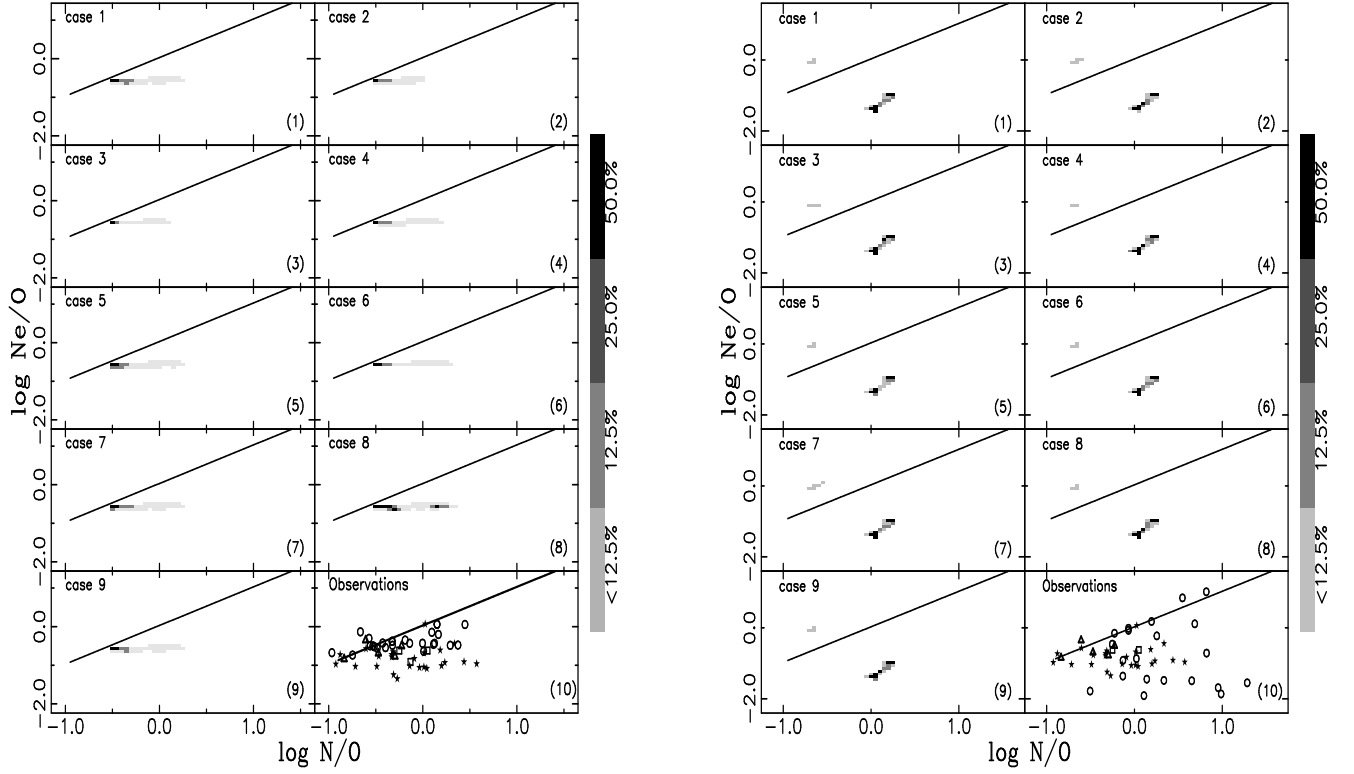


Fig. 4.— Gray-scale maps of the chemical abundance ratios of $\log \text{Ne}/\text{O}$ versus $\log \text{N}/\text{O}$. Solid lines mean $\log \text{Ne}/\text{O} = \log \text{N}/\text{O}$. The left and right panels of Fig. 4 show the chemical abundances of the giants during the symbiotic stage and the ejected materials from the hot companions during the symbiotic novae, respectively. In the panels of the observations, the filled big-stars, the filled triangles and the squares represent the observational values of the symbiotic nebulae in Luna & Costa (2005), those of 5 symbiotic nebulae in De Freitas Pacheco & Costa (1992) and those of V1016 Cyg and HM Sge in Schmid & Schild (1990), respectively. The circles in the left and right panels (10) of Fig. 4 represent the observational values of the planetary nebulae in Pottasch & Bernard-Salas (2006) and the novae in Livio & Truran (1994), respectively.

Schmid & Schild (1990).

Fig. 4 gives the distributions of the abundance ratios of Ne/O vs. N/O in all cases and observations. The left and right panels of Fig. 4 show Ne/O vs. N/O of the cool giants in SSs and the ejected materials from the hot components during symbiotic novae, respectively. The right panel of Fig. 4 shows that our results of the ejected materials from the hot companions during the symbiotic novae are less scattered than the observations of the novae in Livio & Truran (1994). There are mainly two reasons. One reason is too small numerical samples in Kovetz & Prialnik (1997) and José & Hernanz (1998). Another reason is a high uncertainty in observations with a factor of 2-4. In the right panel of Fig. 4, the distributions of Ne/O and N/O are obviously cut into two regions. The left-up region of higher Ne/O originates from the ejected materials from the hot companions during the symbiotic novae with accreting ONe WDs and the middle-down region of lower Ne/O corresponds to the ejected materials from the hot companions during the symbiotic novae with accreting CO WDs.

Panels (10) of Fig. 4 show that the abundance ratios of Ne/O vs. N/O in symbiotic nebulae are basically between those in the planetary nebulae and the novae. Our simulations also support this result. Therefore, the distributions of Ne/O vs. N/O reconfirm that the symbiotic nebulae are modified by the ejected materials from the hot components.

3.5.3. He/H vs. N/O

Luna & Costa (2005) showed the distributions of the observational $\log O/N$ vs. He/H for 43 symbiotic nebulae. The mean error of $\log O/N$ is about 0.1 and the mean error of He/H is within ~ 0.03 . Costa & de Freitas Pacheco (1994) gave those of 5 symbiotic nebulae within an error of 0.038.

As the panels (10) of Fig. 5 show, there are many symbiotic nebulae whose helium abundances are much higher than those of the observational planetary nebulae in Pottasch & Bernard-Salas (2006). The enhancement of the helium abundance in the stellar surface mainly depends on the third dredge-up. However, the left panels (1)–(9) of Fig. 5 show that the He/H ratios in the

stellar surface of the cool giants in all SSs are basically lower than 0.2 in our simulations. The enhancement of the helium abundance in the symbiotic nebulae should originate from the ejected materials during the symbiotic novae. The observational He/H ratios of the classical novae in Livio & Truran (1994) range from 0.07 to 0.40. The observational distribution of He/H vs. $\log N/O$ also support that the symbiotic nebulae are modified by the ejected materials from the hot components. Unfortunately, as the right panels (1)–(9) of Fig. 5 show, our simulations do not offer high He/H ratios for the ejected materials from the hot components. A possible explanation is: if the mass-accretion rates of WD accretors are higher than the critical value \dot{M}_{WD} (See Eq. (24) in Paper I), the nova outbursts are weak and a helium layer may be left on the surface of WD accretors. We call these novae as weak symbiotic novae (Paper I). If the mass-accretion rates of WD accretors are lower than \dot{M}_{WD} , the nova outbursts are so strong that the WDs are eroded. Therefore, no helium layer is left. These novae are called as strong symbiotic novae. A typical symbiotic star can undergo dozens of nova outbursts (Paper I). For the weak symbiotic novae, a helium layer may be left on the surface of WD accretors after each outburst. When the next outburst occurs, the remnant helium can be dredged up and ejected (Iben et al. 1992). In addition, most WDs have been traditionally found to belong to one of the following categories: those with a hydrogen-rich atmosphere (the DAs) and those with a helium-rich atmosphere (the DBs). DA white dwarfs constitute about the 80% of all observed WDs. They have a thin hydrogen layers whose mass is lower than about $10^{-4}M_{WD}$. Also, there is a helium layer (their mass is about $10^{-2}M_{WD}$) under the hydrogen layer (Althaus & Benvenuto 1998). Iben & Tutukov (1985) calculated the distributions of helium, carbon and oxygen through the surfaces layers of a $1.05 M_{\odot}$ DB WD coming from a case B mass-transfer event. They found the amount of helium near the surface is only about $10^{-3}M_{\odot}$. González Pérez & Metcalfe (2007) tested the evolution of the DB white dwarf GD 358. They suggested that binary evolution describes better GD 358 and obtained the helium layer with a mass $10^{-5.66}M_{WD}$. Therefore, when the first nova outbursts occurs, the hydrogen shell

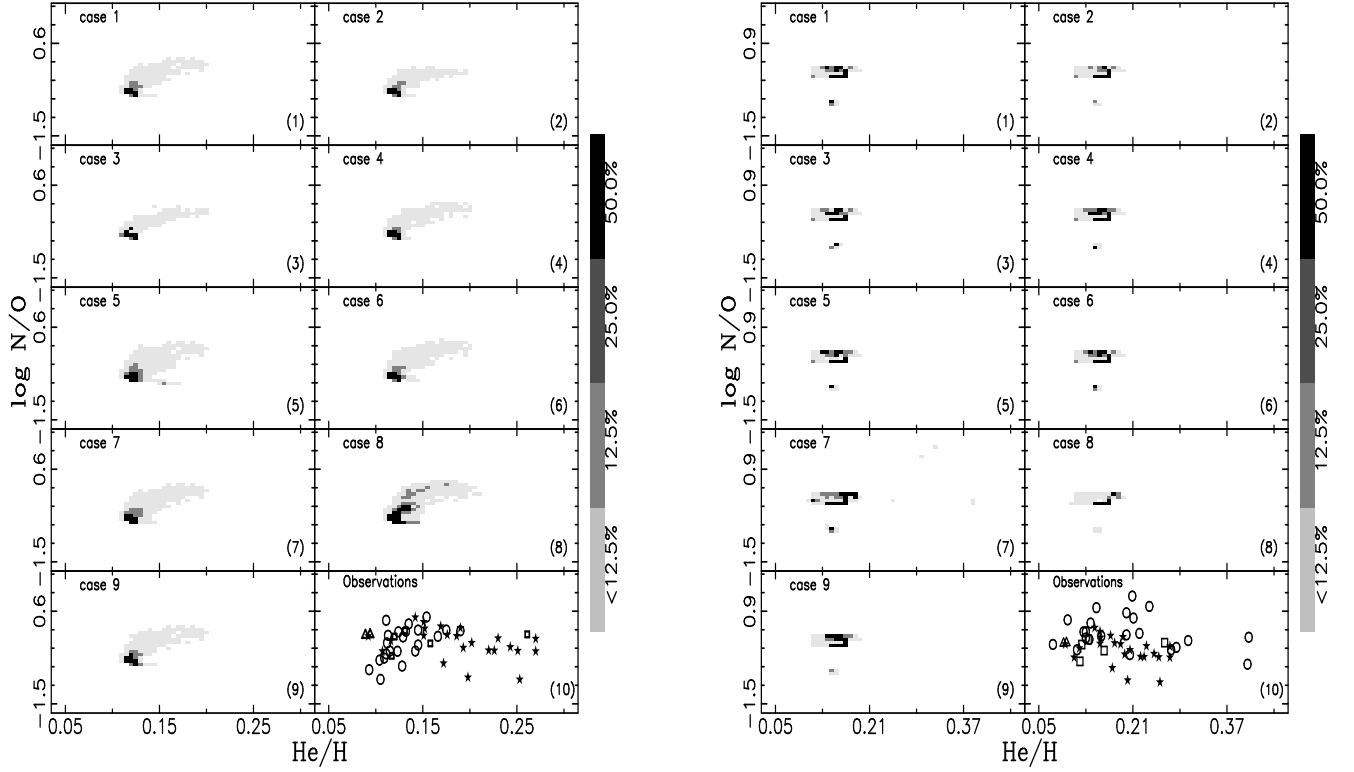


Fig. 5.— Gray-scale maps of the chemical abundance ratios of He/H versus log N/O. The left and right panels of Fig. 5 show the chemical abundances of the giants during symbiotic stage and the ejected materials from the hot companions during the symbiotic novae, respectively. In the panels of the observations, the filled big-stars, the squares and the triangles represent the observational values of the symbiotic nebulae in Luna & Costa (2005), those of 5 symbiotic nebulae in Costa & de Freitas Pacheco (1994) and those of V1016 Cyg and HM Sge in Schmid & Schild (1990), respectively. The circles in the left and right panels (10) of Fig. 5 represent the observational values of the planetary nebulae in Pottasch & Bernard-Salas (2006) and the novae in Livio & Truran (1994), respectively.

flash will penetrate inward to the region where the abundance of H is ~ 0.01 and a certain amount of the helium is dredged up (Iben et al. 1992). In the above situations, helium is overabundant. Our calculations may underestimate helium abundances of the ejected materials from the hot components during the symbiotic novae.

The helium of the symbiotic novae V1016 Cyg and HM Sge in Schmid & Schild (1990) is not overabundant. They may have undergone several strong symbiotic novae so that the WD accretors have no the helium layer. Their helium should be similar to our simulated result.

We suggest that the helium enhancement of the symbiotic nebulae stems from the modification of the ejected materials from the WDs with a helium layer. In our simulations, the ratios of the SSs being in the weak symbiotic novae and their decline phases to total SSs are between 25% (case 2) and 38% (case 9). The helium element of these symbiotic nebulae should be overabundant. The ratio of the symbiotic nebulae whose He/H is higher than 0.2 to the total of the symbiotic nebulae in Luna & Costa (2005) is 33.3%, which are in agreement with our results.

4. Conclusion

We have performed a detailed study of the chemical abundances in SSs and drew several important conclusions.

1. The initial-final mass relation has great effects on SSs' population and especially on the symbiotic novae with massive WD accretors. A steep initial-final mass relation result in an over-estimated occurrence rate of symbiotic novae.

2. The number ratios of O-rich SSs to C-rich SSs in our simulations are between 3.4 and 24.1, and they are sensitive to TDU efficiency λ and the terminal velocity of stellar wind $v(\infty)$. Our simulation may have underestimated the terminal velocity of stellar wind $v(\infty)$ of C-rich giants and the mass-loss rate of the cool giants in SSs. The number ratio of SSs with *extrinsic* C-rich cool giants to all SSs with C-rich cool giants is between 2.1% and 22.7%. The TDU efficiency λ , the common envelope algorithm and the mass-loss rate have a great effect on it.

3. Comparing our $^{12}\text{C}/^{13}\text{C}$ vs. $[\text{C}/\text{H}]$ of the cool giants in SSs with those of the observations,

we infer that thermohaline mixing in low-mass stars should exist. Its effect on the chemical abundances is very significant.

4. The distributions of O/N vs. C/N, Ne/O vs. N/O and He/H vs. N/O of the symbiotic nebulae indicate that it is quite common in SSs that the nebular chemical abundances are modified by the ejected materials from the hot components.

5. Helium abundances in the symbiotic nebulae during the symbiotic novae are determined by whether the WD accretors have a helium layer or not in their surfaces. If they have a helium layer, helium is overabundant. If the WD accretors have only undergone strong symbiotic nova outbursts, they have no helium layer. The helium in the symbiotic nebulae (like V1016 Cyg and HM Sge) are not overabundant.

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REFERENCES

- Althaus L. G., Benvenuto O. G., 1998, MNRAS, 296, 206
- Anders E., Grevesse N., 1989, Grochim. Cosmochim. Acta, 53, 197
- Bedijn P. J., 1988, A&A, 205, 105
- Belczyński K., Mikołajewska J., Munari U., Ivison R. J., Friedjung M., 2000, A&AS, 146, 407
- Bergeat J., Chevallier L., 2005, A&A, 429, 235
- Blöcker T., 1995, A&A, 297, 727
- Bondi H., Hoyle F., 1944, MNRAS, 104, 273
- Boothroyd A. I., Sackmann I. J., 1988, ApJ, 328, 653

- Caughlan G. R., Fowler W. A., 1988, *Atomic Data and Nuclear Data Tables* 40, 283-334.
- Charbonnel C., Zahn J. P., 2007, *A&A*, 467, L15
- Clayton D. D., 1983, *Principles of Stellar EVolution and Nucleosynthesis*. Chicago
- Costa R. D. D., de Freitas Pacheco J. A., 1994, *A&A*, 285, 998
- de Freitas Pacheco J. A., Costa R. D. D., 1992, *A&A*, 257, 619
- Eggleton P. P., Fitechett M. J., Tout C. A., 1989, *ApJ*, 347, 998
- Eggleton P. P., Dearborn D. S. P., Lattanzio J. C., 2006, *Science*, 314, 1580
- Eggleton P. P., Dearborn D. S. P., Lattanzio J. C., 2007, *astro-ph/07062710*
- Girardi L., Bressan A., Bertelli G., Chiosi C., 2000, *A&AS*, 141, 371
- Goldberg D., Mazeh T., 1994, *A&A*, 282, 801
- González Pérez J. M., Metcalfe T., *astro-ph/07083703*
- Gratton R. G., Sneden C., Carretta E., Bragaglia A., 2000, *A&A*, 354, 169
- Groenewegen M. A. T., de Jong T., 1993, *A&A*, 267, 410
- Groenewegen M. A. T., 2002, *Ringberg Castle Workshop (astro-ph/0208449)*
- Han Z., Podsiadlowski P., Eggleton P. P., 1994, *MNRAS*, 270, 121
- Han Z., Eggleton P. P., Podsiadlowski P., Tout C. A., 1995a, *MNRAS*, 277, 1443
- Han Z., Podsiadlowski P., Eggleton P. P., 1995b, *MNRAS*, 272, 800
- Han Z., Eggleton P. P., Podsiadlowski Ph., Tout C. A., Webbink R. F., 2001, *ASP Conf. Ser.*, 229, 205
- Han Z., Podsiadlowski, Ph., Maxted P. F. L., Marsh T. R., Ivanova N., 2002, *MNRAS*, 336, 449
- Hatano K., Branch D., Fisher A., Starrfield S., 1997, *MNRAS*, 290, 113
- Herwig F., 1995, In: Noels A., Fraipont-Caro D., Gabriel M., Grevesse N., Demarque P.(eds.) *Proc. 32. Liege Int. Astrophys. Coll.*, P. 441
- Hurley J. R., Pols O. R., Tout C. A., 2000, *MNRAS*, 315, 543
- Hurley J. R., Tout C. A., Pols R., 2002, *MNRAS*, 329, 897
- Iben I. Jr., Renzini A., 1983, *ARA&A*, 21,271
- Iben I. Jr., Tutukov A. V., 1984, *ApJ*, 282, 615
- Iben I. Jr., Tutukov A. V., 1985, *ApJS*, 58, 661
- Iben I. Jr., Fujimoto M. Y., Macdonald J., 1992, *ApJ*, 388, 521
- Iben I. Jr., Tutukov A. V., 1996, *ApJS*, 105, 145
- Izzard R. G., Tout C. A., Karakas A. I., Pols O. R., 2004, *MNRAS*, 350, 407
- Izzard R. G., 2004, *Doctor thesis, Combrige*
- Jorissen A., 2003, *ASPC*, 303, 25
- José J., Hernanz M., 1998, *ApJ*, 494, 680
- Karakas A. I., Lattanzio J. C., Pols O. R., 2002, *PASA*, 19, 515
- Kenny H. T., Taylor A. R., Seaquist E. R., 1991, *ApJ*, 366, 549
- Kenny H. T., Taylor A. R., 2005, *ApJ*, 619, 527
- Kenny H. T., Taylor A. R., 2007, *ApJ*, 662, 1231
- Kenyon S. J., Webbink R. F., 1984, *ApJ*, 279, 252
- Kenyon S. J., 1986, *The Symbiotic Stars*(Cambridge: Cambridge Univ. Press)
- Kjærgaard P., Gustafsson B., Walker G. A. H., Hultqvist L., 1982, *A&A*, 115, 145
- Kovetz A., Prialnik D., 1994, *ApJ*, 424, 319
- Kovetz A., Prialnik D., 1997, *ApJ*, 477, 356
- Lazaro C., Lynas-Gray A. E., Clegg R. E. S., Mountain C. M., Zdrozny A., 1991, *A&A*, 249, 62

- Livio M., Truran J. W., 1994, *ApJ*, 425, 797
- Luna G. J. M., Costa R. D. D., 2005, *A&A*, 435, 1087
- Lü G. L., Yungelson L., Han Z., 2006, *MNRAS*, 327, 1389
- Marigo P., 2001, *A&A*, 370, 194
- Marigo P., Girardi L., 2007, *A&A*, 469, 239
- Mazeh T., Goldberg D., Duquennoy A., Mayor M., 1992, *ApJ*, 401, 265
- Miller G. E., Scalo J. M., 1979, *ApJS*, 41, 513
- Mikołajewska J., Kenyon S. J., 1992, *MNRAS*, 256, 177
- Mikołajewska, J., 2003, *Astronomical Society of the Pacific Conference Series*, 303, 9
- Mikołajewska, J., 2007, *Baltic Astronomy*, Vol. 16, p. 1-9
- Mitsumoto M., Jahanara B., Matasuda T., et al., 2005, *AREP*, 49, 884
- Mürset U., Nussbaumer H., Schmid H. M., Vogel M., 1991, *A&A*, 248, 458
- Mürset U., Schmid H. M., 1999, *A&A*, 137, 473
- Nauenberg M., 1972, *ApJ*, 175, 417
- Nelemans G., Verbunt F., Yungelson L. R., Portegies Zwart S. F., 2000, *A&A*, 360, 1011
- Nelemans G., Tout C. A., 2005, *MNRAS*, 356, 753
- Nelson L. A., MacCannell K. A., Dubeau E., 2004, *ApJ*, 602, 938
- Nussbaumer H., Schild H., Schmid H. M., Vogel M., 1988, *A&A*, 198, 179
- Nussbaumer H., Vogel M., 1989, *A&A*, 213, 137
- Nussbaumer H., Vogel M., 1990, *A&A*, 236, 117
- Olofsson H., González Delgado D., Kerschbaum F., Schöier F. L., 2002, *A&A*, 391, 1053
- Paczyński B., Rudak B., 1980, *A&A*, 82, 349
- Pols O. R., Schröder K. P., Hurley J. R., Tout C. A., Eggleton P. P., 1998, *MNRAS*, 298, 525
- Pottasch S. R., Bernard-Salas J., 2006, *A&A*, 457, 189
- Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 1992, *Numerical Recipes In Fortran 77. The Art of Scientific Computing Second Edition*. Cambridge Univ. Press, Cambridge
- Prialnik D., Kovetz A., 1995, *ApJ*, 445, 789
- Reimers D., 1975, *Mem. Soc. R. Sci. Liege*, 8, 369
- Schmid H. M., Schild H., 1990, *MNRAS*, 246, 84
- Schild H., Boyle S. J., Schmid H. M., 1992, *MNRAS*, 258, 95
- Schmidt M. R., Mikołajewska J., 2003, *ASPC*, 303, 163
- Schmidt M. R., Zács L., Mikołajewska J., Hinkle K. H., 2006, *A&A*, 446, 603
- Schara M. M., Prialnik D., Kovetz A., 1993, *ApJ*, 406, 220
- Sokoloski J. L., Kenyon S. J., Espey B. R., et al., 2006, *ApJ*, 636, 1002
- Stancliffe R. J., Jeffery C. S., 2007, *MNRAS*, 375, 1280
- Starrfield S., Truran J. W., Wiescher M. C., Sparks W. M., 1998, *MNRAS*, 296, 502
- Tutukov A. V., Yungelson L. R., 1976, *Astrophysics*, 12, 321
- van den Hoek L. B., Groenewegen M. A. T., 1997, *A&AS*, 123, 305
- Vassiliadis E., Wood P. R., 1993, *ApJ*, 413, 641
- Vogel M., Nussbaumer H., 1992, *A&A*, 259, 525
- Wagenhuber J., Groenewegen M. A. T., 1998, *A&A*, 340, 183
- Whitelock P. A., Munari U., 1992, *A&A*, 255, 171
- Willson L. A., Wallerstein G., Brugel E. W., Stencel R. E., 1984, *A&A*, 133, 254
- Willson L. A., 2007, *astro-ph/07043589*
- Winters J. M., Le Bertre T., Jeong K. S., Helling Ch., Sedlmayr E., 2000, *A&A*, 361, 641

- Winters J. M., Le Bertre T., Jeong K. S., Nyman L.-Å., Epchtein N., 2003, A&A, 409, 715
- Weidemann V., 2000, A&A, 363, 647
- Yaron O., Prialnik D., Shara M. M., Kovetz A., 2005, ApJ, 623, 398
- Yungelson L., Livio M., Tutukov A. V., Saffer R. A., 1994, ApJ, 420, 336
- Yungelson L., Livio M., Tutukov A. V., Kenyon S. J., 1995, ApJ, 477, 656

A. Synthesis TP-AGB Evolution

Stellar evolution from the zero age main sequence up to the first thermal pulse is dealt with in the rapid evolution code of Hurley et al. (2000). The changes of the chemical abundances on the stellar surface during the giant branch phase (the first dredge up) and early asymptotic giant branch phase (the second dredge up) can be represented by simple fitting formulae in Izzard et al. (2004) and Izzard (2004). After the first thermal pulse, we use a synthetic model for TP-AGB.

A.1. The initial abundances

The initial abundances (*i.e.* of zero age main sequence stars) are taken from Anders & Grevesse (1989) for $Z = 0.02$. The following shows the initial abundances by mass fractions:

$$\begin{aligned} {}^1\text{H} &= 0.68720; & {}^4\text{He} &= 0.29280; \\ {}^{12}\text{C} &= 2.92293 \times 10^{-3}; & {}^{13}\text{C} &= 4.10800 \times 10^{-5}; \\ {}^{14}\text{N} &= 8.97864 \times 10^{-4}; & {}^{15}\text{N} &= 4.14000 \times 10^{-6}; \\ {}^{16}\text{O} &= 8.15085 \times 10^{-3}; & {}^{17}\text{O} &= 3.87600 \times 10^{-6}; \\ {}^{20}\text{Ne} &= 2.29390 \times 10^{-3}; & {}^{22}\text{Ne} &= 1.45200 \times 10^{-4}. \end{aligned}$$

A.2. Core mass at the first thermal pulse

Using the rapid evolution code of Hurley et al. (2000), we can obtain the stellar mass ($M_{1\text{TP}}$) at the first thermal pulse. The core mass at the first thermal pulse, $M_{c,1\text{TP}}$ is taken from Karakas, Lattanzio & Pols (2002):

$$M_{c,1\text{TP}} = [-p_1(M_0 - p_2) + p_3]f + (p_4M_0 + p_5)(1 - f), \quad (\text{A1})$$

where $f = (1 + e^{(\frac{M_0 - p_6}{p_7})})^{-1}$, M_0 is the initial mass in solar unit and the coefficients $p_1, p_2, p_3, p_4, p_5, p_6$ and p_7 are in Table 6 of Karakas, Lattanzio & Pols (2002).

A.3. Luminosity, radius, interpulse period, and evolution of core mass

We use the prescriptions of Izzard et al. (2004). The luminosity is taken as the value calculated from Eq.(29) in Izzard et al. (2004). We define the radius R as $L = 4\pi\sigma R^2 T_{\text{eff}}^4$, where σ is the Stefan-Boltzmann constant and T_{eff} is the effective temperature of the star. The radius is taken the value calculated from Eq.(35) in Izzard et al. (2004). The interpulse period τ_{ip} is:

$$\log_{10}(\tau_{\text{ip}}/\text{yr}) = a_{28}(M_c/M_\odot - b_{28}) - 10^{c_{28}} - 10^{d_{28}} + 0.15\lambda^2, \quad (\text{A2})$$

where the third dredged-up (TDU) efficiency λ is defined in §A.4, and the coefficients are:

$$a_{28} = -3.821,$$

$$b_{28} = 1.8926,$$

$$c_{28} = -2.080 - 0.353Z + 0.200(M_{\text{env}}/M_\odot + \alpha - 1.5),$$

$$d_{28} = -0.626 - 70.30(M_{c,1\text{TP}}/M_\odot - \zeta)(\Delta M_c/M_\odot),$$

where M_{env} represents the envelope mass, α is the mixing length parameter and equals to 1.75, $\zeta = \log(Z/0.02)$, and ΔM_c is the change in core mass defined as $\Delta M_c = M_c - M_{c,1\text{TP}}$.

A.4. The minimum core mass for TDU and the TDU efficiency

TDU can occur only for stars above a certain core mass M_c^{min} . Groenewegen & de Jong (1993) took M_c^{min} as a constant 0.58. Karakas, Lattanzio & Pols (2002) found that M_c^{min} depends on stellar mass and metallicity and gave a fitting formula by

$$M_c^{\text{min}} = a_1 + a_2M_0 + a_3M_0^2 + a_4M_0^3, \quad (\text{A3})$$

where coefficients a_1 , a_2 , a_3 and a_4 are shown in Table 7 of Karakas, Lattanzio & Pols (2002) and M_0 is the initial mass in solar unit. According to the observed carbon luminosity function in the Magellanic clouds, Marigo & Girardi (2007) considered that the M_c^{\min} predicted by Karakas, Lattanzio & Pols (2002) is high. In this work, we carry out various numerical simulations (See Table 1).

(i) Like Groenewegen & de Jong (1993), $M_c^{\min}=0.58M_\odot$;

(ii) Following Karakas, Lattanzio & Pols (2002), we take Eq. (A3) as M_c^{\min} ;

It should be recalled that $M_c^{\min}=M_{c,1TP}$ if stellar initial mass $M_{\text{initial}} \geq 4M_\odot$ (Karakas, Lattanzio & Pols 2002; Izzard et al. 2004) or $M_c^{\min} < M_{c,1TP}$.

The TDU efficiency is defined by $\lambda = \frac{\Delta M_{\text{dred}}}{\Delta M_c}$, where ΔM_{dred} is the mass brought up to the stellar surface during a thermal pulse. λ is a very uncertain parameter. Karakas, Lattanzio & Pols (2002) showed a relation of λ :

$$\lambda(N) = \lambda_{\max}[1 - \exp(-N/N_r)], \quad (\text{A4})$$

where λ gradually increases towards an asymptotic λ_{\max} with N (the progressive number of thermal pulsation) increasing. N_r is taken from Eq. (49) in Izzard et al. (2004) which reproduces the results for N_r in Table 5 of Karakas, Lattanzio & Pols (2002). λ_{\max} is given by (See Eq.(6) in Karakas, Lattanzio & Pols (2002)):

$$\lambda_{\max} = \frac{b_1 + b_2 M_0 + b_3 M_0^3}{1 + b_4 M_0^3}, \quad (\text{A5})$$

where coefficients are shown in Table 8 of Karakas, Lattanzio & Pols (2002). Groenewegen & de Jong (1993) took λ as a constant 0.75. In order to test the influences of λ on our results, we carry out numerical simulations with various λ (See Table 1).

(i) Following Karakas, Lattanzio & Pols (2002), $\lambda=\text{Eq. (A4)}$;

(ii) Following Groenewegen & de Jong (1993), $\lambda=0.75$;

(iii) Simulating a small TDU efficiency, $\lambda=0.5$.

A.5. Inter-shell abundances

During every thermal pulse, the dredged mass ΔM_{dred} mixes into stellar envelope. Based on the nucleosynthesis calculations by Boothroyd & Sackmann (1988), Marigo & Girardi (2007) gave the fit of the abundances of ^4He , ^{12}C and ^{16}O in the inter-shell region:

For $\Delta M_c \leq 0.025M_\odot$:

$$^{\text{inter}}X(^4\text{He}) = 0.95 + 400(\Delta M_c)^2 - 20.0\Delta M_c$$

$$^{\text{inter}}X(^{12}\text{C}) = 0.03 - 352(\Delta M_c)^2 + 17.6\Delta M_c$$

$$^{\text{inter}}X(^{16}\text{O}) = -32(\Delta M_c)^2 + 1.6\Delta M_c.$$

For $\Delta M_c > 0.025M_\odot$:

$$^{\text{inter}}X(^4\text{He}) = 0.70 + 0.65(\Delta M_c - 0.025)$$

$$^{\text{inter}}X(^{12}\text{C}) = 0.25 - 0.65(\Delta M_c - 0.025)$$

$$^{\text{inter}}X(^{16}\text{O}) = 0.02 - 0.065(\Delta M_c - 0.025).$$

However, there is some debate of the exact composition in the inter-shell region. Izzard et al. (2004) gave some fits of inter-shell abundance, in which inter-shell elements include ^4He , ^{12}C , ^{16}O and ^{22}Ne . Details can be seen from §3.3.3 in Izzard et al. (2004). Their models did not obtain high values of inter-shell ^{16}O such as 2% reported by Boothroyd & Sackmann (1988). The typical inter-shell abundances ($5 M_\odot$, $Z = 0.02$) are $^4\text{He}=0.74$, $^{12}\text{C}=0.23$, $^{16}\text{O}=0.005$ and $^{22}\text{Ne}=0.02$.

In our work, we carry out simulations with various inter-shell abundances.

A.6. The hot bottom burning

If the hydrogen envelope of an AGB star is sufficiently massive, the hydrogen burning shell can extend into the bottom of the convective region. This process is called as hot bottom burning(HBB). For HBB,

we use a treatment similar to that in Groenewegen & de Jong (1993). For the model of Iben & Renzini (1983), Groenewegen & de Jong (1993) gave the most suitable parameter for the fraction of newly dredged up matter exposed to the high temperatures at the bottom of envelope $f_{\text{HBB}} = 0.94$, the fraction of envelope matter mixed down to the bottom of the envelope $f_{\text{bur}} = 3 \times 10^{-4}$, and the exposure time of matter in the region of HBB $t_{\text{HBB}} = 0.0014\tau_{\text{ip}}$. The temperature at the bottom of convective envelope T_{bce} is given by Eq. (37) of Izzard et al. (2004).

For the dredged up mass, the amounts of material added to the envelope are:

$$\begin{aligned}\Delta^4\text{He} &= {}^{\text{inter}}X_4\Delta M_{\text{dred}} \\ \Delta^{12}\text{C} &= [(1 - f_{\text{HBB}})^{\text{inter}}X_{12} \\ &\quad + \frac{f_{\text{HBB}}}{t_{\text{HBB}}} \int_0^{t_{\text{HBB}}} X_{12}^{\text{HBB}}(t)dt]\Delta M_{\text{dred}} \\ \Delta^{13}\text{C} &= [\frac{f_{\text{HBB}}}{t_{\text{HBB}}} \int_0^{t_{\text{HBB}}} X_{13}^{\text{HBB}}(t)dt]\Delta M_{\text{dred}} \\ \Delta^{14}\text{N} &= [\frac{f_{\text{HBB}}}{t_{\text{HBB}}} \int_0^{t_{\text{HBB}}} X_{14}^{\text{HBB}}(t)dt]\Delta M_{\text{dred}} \\ \Delta^{16}\text{O} &= [(1 - f_{\text{HBB}})^{\text{inter}}X_{16} \\ &\quad + \frac{f_{\text{HBB}}}{t_{\text{HBB}}} \int_0^{t_{\text{HBB}}} X_{16}^{\text{HBB}}(t)dt]\Delta M_{\text{dred}},\end{aligned}\tag{A6}$$

where $X^{\text{HBB}}(t)$ are calculated by the way of Clayton's CNO bicycle (Clayton 1983). All details of Clayton's CNO bicycle can be seen from Clayton (1983); Groenewegen & de Jong (1993); Izzard et al. (2004). The CNO bicycle can be split into CN cycle and ON cycle. The timescales in ON cycle are many thousands of times longer than those required to bring the CN cycle into equilibrium. Even in the most massive AGB stars undergoing vigorous HBB, the ON cycle never approaches equilibrium. Therefore, the effects of HBB mainly turn ^{12}C into ^{14}N and the abundance of ^{16}O is not changed much. In calculating Clayton's CNO bicycle, the density at the base of the convective envelope is given by Eq. (42) of Izzard et al. (2004) and the analytic expressions for the nuclear reaction rates in Clayton's CNO bicycle are taken from Caughlan & Fowler (1988). The initial conditions of $X^{\text{HBB}}(t)$ are $X^{\text{HBB}}(t=0) = {}^{\text{inter}}X$ for ^{12}C and ^{16}O , while $X^{\text{HBB}}(t=0) = 0$ for ^{13}C and ^{14}N .

After every thermal pulse, the chemical abundances of stellar envelope X^{new} are

$$X^{\text{new}} = \frac{X^{\text{old}}M_{\text{env}}(1 - f_{\text{bur}}) + \Delta X + \frac{f_{\text{bur}}M_{\text{env}}}{t_{\text{HBB}}} \int_0^{t_{\text{HBB}}} X(t)dt}{M_{\text{env}} + \Delta M_{\text{dred}}},\tag{A7}$$

where ΔX is given by Eq. (A6) and the initial conditions of $X(t)$ are $X(t=0) = X^{\text{old}}$.

A.7. Mass loss

The mass-loss rate of the cool giant during AGB phase has a great effect on the population of SSs and the chemical evolution of the stellar surface. We consider two laws of the mass-loss rate in this work.

(i) A mass-loss relation suggested by Vassiliadis & Wood (1993) based on the observations, given as

$$\log \dot{M} = -11.4 + 0.0123(P - 100 \max(M/M_{\odot} - 2.5, 0.0)),\tag{A8}$$

where P is the Mira pulsation period in days given by

$$\log P = -2.07 + 1.94 \log(R/R_{\odot}) - 0.90 \log(M/M_{\odot}).\tag{A9}$$

When $P \geq 500$ days, the steady super-wind phase is modeled by the law

$$\dot{M}(M_{\odot}\text{yr}^{-1}) = 2.06 \times 10^{-8} \frac{L/L_{\odot}}{v_{\infty}},\tag{A10}$$

where v_{∞} is the terminal velocity of the super-wind in km s^{-1} . We use $v_{\infty} = 15 \text{ km s}^{-1}$ in this paper.

(ii) Based on the simulations of shock-driven winds in the atmospheres of Mira-like stars in Bedijn (1988), Blöcker (1995) gave a mass-loss rate similar to Reimers' formula:

$$\dot{M} = 4.83 \times 10^{-9} M^{-2.1} L^{2.7} \dot{M}_{\text{Reimers}},\tag{A11}$$

where \dot{M}_{Reimers} is given by Eq.(A12) but $\eta = 0.02$ (Stancliffe & Jeffery 2007).

On other stellar evolutionary phase, the mass-loss rates are given by Reimers' formula (Reimers 1975):

$$\dot{M} = -4.0 \times 10^{-13} \eta \frac{LR}{M} \text{M}_{\odot} \text{yr}^{-1}, \quad (\text{A12})$$

where L , R , M are the stellar luminosity, radius and mass in solar units and we use $\eta=0.5$.

A.8. Comparison with previously published yields by other models

In order to test our synthetic model of asymptotic giant branch stars, we compare the stellar yields in our model with previously published yields of van den Hoek & Groenewegen (1997), Marigo (2001) and Izzard et al. (2004). According to the definition of stellar yield in Izzard et al. (2004), we write the total yield of isotope j by

$$Y_j = \int_0^{\tau(M_i)} (X_j - X_j^0) \frac{dM}{dt} dt, \quad (\text{A13})$$

where $\tau(M_i)$ is the entire lifetime of a star with initial mass M_i , $\frac{dM}{dt}$ is the current mass-loss rate, X_j and X_j^0 refer to the current and initial surface abundance of the isotope j , respectively.

Figure 6 shows the stellar yields of van den Hoek & Groenewegen (1997) [for the models of $Z = 0.02$, $\eta_{\text{AGB}} = 4.0$ and $m_{\text{HBB}} = 0.8$ (See Table 17 of van den Hoek & Groenewegen 1997)], Marigo (2001) (for the models of $Z = 0.019$ and the mixing length parameter $\alpha = 1.68$), Izzard et al. (2004) [for the models of $Z = 0.02$ (See Table D2 of Izzard et al. 2004)] and ours. In our models, all physical parameters are the same as those in case 1 except that we change binary systems into single star. As Figure 6 shows, the stellar yields of our models in general are in agreement with those in Izzard et al. (2004) or lie between those of van den Hoek & Groenewegen (1997) and Izzard et al. (2004) although our models produce more ^{15}N , and less ^{17}O at massive stars.

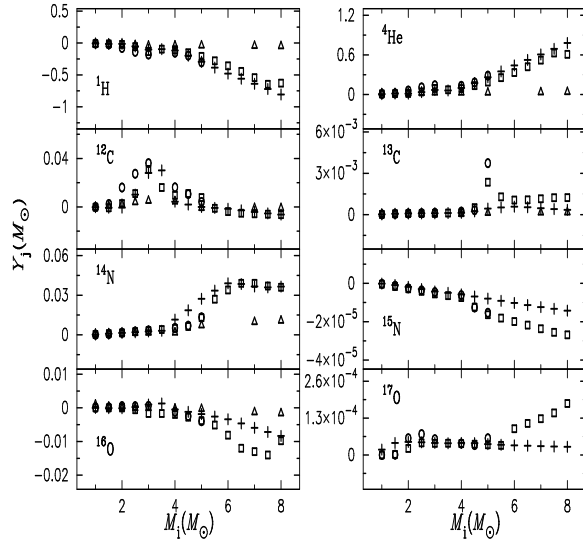


Fig. 6.— Total stellar yields vs. the initial stellar masses. Triangles, circles and squares represent the models of van den Hoek & Groenewegen (1997), Marigo (2001) and Izzard et al. (2004), respectively. Pluses are our results.